Firm Export Dynamics in Interdependent Markets*

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Abstract

We estimate a model of firm export dynamics featuring cross-country complementarities. The firm decides where to export by solving a dynamic combinatorial discrete choice problem, for which we develop a solution algorithm that overcomes the computational challenges inherent to the large dimensionality of its state space and choice set. According to our estimated model, firms enjoy cost reductions when exporting to countries geographically or linguistically close to each other, or that share deep trade agreements. Countries, especially small ones, sharing these traits with attractive destinations receive significantly more exports than in the absence of complementarities.

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1 Introduction

Since Baldwin (1988), a large literature documents the dynamics of firms in export markets. This literature nearly unanimously assumes a firm's exports to a country are unaffected by its exports to other countries. There is however a growing body of evidence questioning this assumption, supporting instead the hypothesis that there are cross-country export complementarities, understood as mechanisms through which a firm's export participation in a country (due, for example, to low tariffs) makes the firm more likely to export to other countries. The current literature nonetheless lacks a quantification of the role these complementarities play in the firm's export choices. In this paper, we take a first step towards this quantification.

Quantifying the importance of cross-country complementarities while accounting for the dynamics of firms in export markets requires addressing two challenges. First, one must identify the impact that the firm's decision to export to a country has on its export profits in other countries. Here, we follow a literature (e.g., Albornoz et al., 2021; Mattoo et al., 2022) that uses instruments to separately identify export complementarities from cross-market correlation in unobserved determinants of export choices, addressing in this way an identification challenge similar to the reflection problem studied in Manski (1993). Second, one must devise an algorithm to solve dynamic firm entry models with cross-country complementarities. This is our main contribution. Building on Jia (2008) and Arkolakis et al. (2023), we design an algorithm to solve rational-expectations single-agent dynamic discrete choice models with cross-choice complementarities. Our algorithm addresses the computational challenges inherent to the large dimensionality of the state space and choice set in this kind of models. The algorithm is applicable to a broad set of dynamic discrete choice models with supermodular objective functions, as illustrated, e.g., in Liu (2023).

The notion of cross-country complementarities features prominently in trade policy discussions. Specifically, it is often used as a justification for the signing of preferential trade agreements (PTAs), which interested parties proclaim act as gateways to markets beyond those of the agreements' signatories.[1] Relatedly, in the specific case of deep PTAs, the belief that the regulatory convergence these treaties impose on their members is itself a source of complementarities between them has been used to argue that these agreements help attract imports from third countries. This line of reasoning was particularly salient in policy discussions of Brexit.[2] Using our model and solution algorithm, we evaluate the impact of

[1] To provide an example involving Costa Rica, whose data we use in this paper, its government defended its 2013 PTA with Singapore by claiming it would increase its exports all throughout Asia (Ruiz, 2013).

[2] Models à la Eaton and Kortum (2002) or Anderson and van Wincoop (2003) predict that shallow PTAs divert trade from third countries. When forecasting the impact of Brexit, UNCTAD (2020) hypothesized this

counterfactual trade policies with the aim of shedding light on these policy discussions.

We extend a canonical model of firm export dynamics à la Das et al. (2007) to allow for cross-country complementarities. Our model features monopolistically competitive firms with constant marginal production costs and country- and period-specific variable, fixed, and sunk export costs. Variable costs are "iceberg," and firms face sunk costs when exporting to countries to which they did not export in the previous period (see Roberts and Tybout, 1997). All export costs in a country are allowed to depend on its geographic and linguistic distance to, and the deepness of its PTAs with, the firm's home country. The fixed cost a firm faces in a country and period may additionally depend on the firm's other export destinations in the same period. Specifically, a firm may face smaller fixed costs in a country if it concurrently exports to another country, and the size of this cost reduction may depend on these countries' geographic and linguistic proximity as well as the deepness of the PTAs of which both are members. To discipline the estimation of the parameters determining the extent to which fixed costs in a country depend on the firm's export choices in other countries, we allow these costs to also depend on a term unobserved to the researcher that is potentially correlated across countries.

The inclusion of sunk costs, and our modeling of fixed costs, make a firm's static export profits in a country and period weakly larger if the firm exported to the same country in the previous period, or if it exports to other countries in the same period. The firm internalizes the impact its export choice in a country and period has on profits in other countries and periods. It thus chooses each period's set of export destinations as the solution to a dynamic combinatorial discrete choice problem. When forecasting the impact of its choices on future periods, firms form expectations rationally.

Given commonly available computational capabilities, the optimization problem determining the firm's export path cannot be solved using standard algorithms. The reason is that, given J foreign countries, the choice set in any given period includes 2^J elements (each one a J-dimensional vector of binary variables indicating the set of countries to which the firm exports), and the state space similarly grows exponentially in the number of possible destinations. To compute the firm's optimal export path, we develop an algorithm that solves a series of increasingly complex problems that place gradually tighter bounds on the firm's optimal choice. Our algorithm exploits the supermodularity of the firm's objective function: exporting to a country in a period and state weakly increases the returns to exporting in every other country, future period, and possible state. It thus builds on prior work that has leveraged the supermodularity of the objective function to solve otherwise

negative third-country effect would be mitigated by increased UK-EU regulatory divergence as "trade costs rise for third countries due to production process adjustment costs or duplication of proofs of compliance."

intractable static optimization problems (see Jia, 2008, Antràs et al., 2017, Arkolakis et al., 2023), and it extends the set of problems that are computationally feasible to solve to a family of supermodular problems featuring dynamics and firms' uncertainty about future payoffs. In our implementation, for faster computation of the solution to the firm's problem, we assume the firm has perfect information on the future path of some payoff-relevant variables. However, this informational assumption is not required for the validity our solution algorithm, and can be relaxed at the expense of computation time.

The problem of separately identifying the parameters governing the strength of the export complementarities from those determining the cross-country correlation in fixed costs' unobserved determinants is an instance of the general problem of separately identifying "path" (or group) dependence from correlated unobservables; in our case, across countries within a period. Given any proximity measure between countries, be it geographic or linguistic, or whether they share a deep PTA, we use two types of moments to separately identify these parameters. First, moments capturing how the covariance in firm export choices in any two countries depends on their proximity. Second, moments capturing how the probability the firm exports to a destination depends on exogenous shifters of export profits in *other* countries close to it. These shifters operate in our estimation as instruments for the firm's export participation in those other countries.[3] While the first type of moments is particularly sensitive to the parameters determining the correlation in unobserved fixed cost shocks, the second one is especially sensitive to the parameters determining the impact exporting to a country has on fixed costs in other countries. We present simulation results illustrating that, in our model, both types of moments jointly identify the parameters of interest.

Our estimates reveal a large heterogeneity across country pairs in the impact exporting to one of them has on fixed costs in the other one. This heterogeneity reflects their geographic and linguistic proximity, as well as the deepness of the PTAs tying together their regulations; e.g., exporting to Korea reduces fixed costs in China in 0.3%, exporting to Canada brings down costs in the US in 3.5%, and exporting to France reduces costs in Germany in 9%. These cost savings accumulate as the firm adds destinations; e.g., for a firm exporting to France, adding Switzerland to its export bundle increases the reduction in fixed costs in Germany from 9% to 16%. Generally, EU members, being geographically close to each other and sharing a deep PTA, have fixed costs that are particularly sensitive to the firm export choices in the other members.

[3]We use the aggregate export potential of a country in a sector as an export profit shifter for firms in that sector. We measure a country's export potential as the importer fixed effect in a sectoral gravity equation estimated using data on all country pairs that do not include Costa Rica as importer or exporter. A limitation of this strategy is that the destination-specific profit shifters vary not by firm but by sector. For a similar identification approach that uses product-specific export profit shifters, see Albornoz et al. (2021).

We perform three types of analysis. First, to quantify the role complementarities play in determining firm exports, we compare the predictions of a version of our model in which we set to zero the parameters determining the strength of complementarities to those of alternative versions in which some or all of these parameters take their estimated values. Overall, complementarities increase the number of firm-country-periods with positive exports in 12%, and total exports in 5%. Of the three sources of complementarities we consider, geographical proximity plays the largest role, causing a 3% increase in exports, while deep PTAs generate complementarities that increase exports in 2%, and linguistic proximity does so in 1%. These numbers mask a large heterogeneity across countries: while EU members see Costa Rican exports increase in at least 10%, with countries in Central Europe experiencing increases above 25%, exports to countries such as the US or China are largely unaffected.

Solving our estimated model takes 125 times more than solving the version with no complementarities. There is thus a trade-off between computation time and accuracy of the model predictions. To explore this trade-off, we compute the predictions of versions of our model in which we group countries into clusters such that complementarities between countries in the same cluster are set to their estimated values, and those between countries in different clusters are set to zero. When equating clusters to continents, the time required to solve the model decreases in 30%, and we find that within-continent complementarities increase the number of firm-country-periods with positive exports in 10%, and total exports in 4%. When partitioning countries into 50 clusters according to the estimated complementarities, the computation time decreases in 75%, and the implied increase in export events and total exports, relative to the model with no complementarities, equals 9% and 3%, respectively. Intuitively, as spatial proximity is the key source of complementarities according to our estimates, and the estimated geographical complementarities decrease rapidly in the distance between countries, the predictions of our model are well approximated by those of alternative models that partition the set of countries into a large number of small geographical clusters.

Second, to shed light on discussions on the third-country effect of complementarities due to deep PTAs, we quantify the impact of Brexit on Costa Rican exports to the UK and the EU. Specifically, we compare model-predicted exports in a setting in which the UK and the EU share no deep PTA post Brexit to those in a scenario in which the UK still belongs to the EU and, thus, shares a deep PTA with its members. Trade barriers between Costa Rica and all destinations are kept the same in both scenarios. Given the partial-equilibrium nature of our model, this analysis isolates the third-country effect of Brexit due to cross-country complementarities alone. We find that, in the four years between the Brexit referendum and the UK withdrawal from the EU, firms anticipate the future reduction in UK-EU complementarities, causing the number of firm-periods with positive exports and total exports to the UK to decrease in 1.6% and 0.8%, respectively. In the ten years following the withdrawal, the number of firm-periods with positive exports and total exports to the UK drop in close to 5%. Conversely, the impact on exports to the EU is minimal.

Third, and last, we study the impact of Costa Rica signing PTAs that bring its export tariffs with different destinations to zero, and compare our model's predictions to those of a re-estimated model that rules out the possibility of complementarities. Our model predicts eliminating Costa Rican export tariffs with the EU increases the number of firm-country-years with positive exports and total exports to its members in 65% and 83%, respectively. Although tariffs with non-EU countries do not change, exports to some of them are affected; e.g., exports to Iceland and to the UK increase in close to 7%, reflecting that the former shares a deep PTA with the EU, and the latter is geographically close to several of its members. The model without complementarities predicts smaller increases of 55% and 80% in export participation and total exports to EU members, respectively, and no change in exports to non-members. When eliminating tariffs with CPTPP members instead, our model predicts an increase in exports to these countries that is less than 1 pp. higher than that predicted by the model without complementarities, and no significant change in exports to non-member countries. The reason for the larger divergence in model predictions when studying a change in trade policy with EU members than when doing so with CPTPP members is that the former exhibit stronger complementarities among themselves and with non-member countries than the latter. Thus, whether models that allow for complementarities yield predictions similar to those of models that do not depends on the policy change being studied.

Our results have implications for the right definition of export markets. Customs agencies generally provide information on export destinations by foreign country. Researchers thus typically equate foreign countries to markets. However, for firms, countries and markets may not be synonyms. For example, a firm from Costa Rica selling in California may face non-zero fixed costs when adding clients in Massachusetts, but an exporter selling in Germany may find that the extra fixed costs of adding Austria as a destination are minimal. While data limitations complicate studying whether large countries are better approximated as aggregations of markets, our analysis helps determine whether countries sharing geographical, linguistic, or regulatory, similarities are better treated as single markets. For example, our results reveal that the EU is far from being a collection of 27 independent markets.[4]

Our model assumes away the presence of substitutabilities in the firm's export choices across destinations. The reason is computational, as our algorithm is guaranteed to find the solution to the firm's problem only when the value function is supermodular. Although

[4]For an analysis of market aggregation and the gravity equation, see Redding and Weinstein (2019).

factors that make the firm's sales in different spatial markets substitutable have been shown to be relevant in other settings (Almunia et al., 2021; Boehm and Pandalai-Nayar, 2022), and may also be present in ours, our estimates suggest that complementarities are the dominant force in our data. In our estimation, the moment that identifies the parameters that determine the strength of the cross-country complementarities is monotonic in these parameters. Thus, our estimate of these parameters would have equaled zero if it had been the case that firms export decisions across countries exhibit subsitutabilities in net.

Our paper relates to several strands of the literature. First, it relates to the literature on export dynamics which, as reviewed in Alessandria et al. (2021a), has largely studied the firm's export decision in an aggregate market (Roberts and Tybout, 1997; Das et al., 2007; Alessandria and Choi, 2007; Arkolakis, 2016; Ruhl and Willis, 2017) or in independent markets (Fitzgerald et al., 2023).[5] Exceptions are Schmeiser (2012), Chaney (2014), Albornoz et al. (2016), and Morales et al. (2019), which allow for cross-market firm export complementarities. Relative to this work, our contribution is twofold: first, we solve a canonical partial-equilibrium model of firm export dynamics extended to allow for complementarities across many markets; second, we use the estimated model to quantify the role complementarities play in determining the reaction of firm exports to policy changes.[6]

Second, our paper also relates to a reduced-form literature identifying cross-market interdependencies in firm exports. While there is a large literature documenting correlation patterns in firm sales across markets (Lawless, 2009; Albornoz et al., 2012, 2023), there is a more recent literature using instruments to separately identify cross-market interdependencies from correlation in unobserved determinants of firm sales (Defever et al., 2015; Berman et al., 2015; Albornoz et al., 2021; Mattoo et al., 2022).[7] Our contribution is to allow for complementarities in an export dynamics model, and to quantify the role complementarities play both in overall firm exports and in their reaction to changes in trade policy.

Third, our paper relates to the work solving combinatorial discrete choice problems. This literature has focused nearly exclusively on static problems, and has implemented several approaches: evaluating all choices (Tintelnot, 2017); modeling combinatorial choices as an

[5] Other work on export dynamics in a single market or independent markets includes Eaton et al. (2008, 2021a,b); Alessandria and Choi (2014a,b); Albornoz et al. (2016); Fitzgerald and Haller (2018); Dickstein and Morales (2018); Gumpert et al. (2020); Alessandria et al. (2021b). Work on dynamics in imports or multinational production with independent markets includes Conconi et al. (2016); Ramanarayanan (2017); Garetto et al. (2021); Lu et al. (2022).

[6] The paper closest to ours is Morales et al. (2019), which partially identifies export complementarities under weak restrictions on firm expectations, choice sets, and planning horizons, without solving the resulting model. Hoang (2022) uses a similar methodology to Morales et al. (2019) to identify complementarities in firm imports. In Section 6, we compare our estimates to those in Morales et al. (2019)

[7] There is also a literature studying complementarities between exporting and importing in a market; e.g., Kashara and Lapham (2013) or Antràs et al. (2017).

aggregation of multinomial ones (Hendel, 1999); approximating the discrete problem as a choice over a continuous variable (Oberfield et al., 2023; Castro-Vincenzi, 2022); using simulation-based global optimization algorithms that converge to the solution as the number of simulations grows to infinity (Houde et al., 2023; Castro-Vincenzi et al., 2023); or, devising algorithms that exploit the super- or sub-modularity of the objective function (Jia, 2008; Antràs et al., 2017; Arkolakis et al., 2023).[8] Building on this last approach, we introduce an algorithm to solve rational-expectations single-agent dynamic discrete choice problems in which all choices are complements.

The rest of the paper proceeds as follows. Section 2 describes our data and documents correlation patterns in firm exports. Section 3 introduces our model, and sections 4 and 5 explain how we solve and estimate it, respectively. In Section 6, we present the model estimates, and we discuss counterfactual results in Section 7. Section 8 concludes.

2 Data

In Section 2.1, we describe the sources of our data and present summary statistics. In Section 2.2, we discuss correlation patterns in firm export choices across destinations.

2.1 Sources and Summary Statistics

Our analysis uses two types of data: data on characteristics of firms located in Costa Rica, and data on characteristics of foreign countries as destinations of Costa Rican exports.

Our firm-level data covers the period 2005-2015 and comes from three sources. First, the Costa Rican customs database, which provides information on export revenues by firm, foreign country, and year for the universe of Costa Rican firms. Second, an administrative dataset that, for all firms located in Costa Rica, contains information on their sector, total sales, and expenditure in labor and materials. Using information in these datasets, we construct a measure of firm domestic sales by subtracting total export revenue from total sales. Third, a dataset built by Alfaro-Ureña et al. (2022), which identifies the Costa Rican firms that belong to a foreign multinational corporation. We merge the three datasets using firm identifiers provided by Alfaro-Ureña et al. (2022), and restrict our sample to include only manufacturing firms (i.e., whose main activity is in sectors 10 to 33 of the ISIC Rev. 4 classification) that are not part of a foreign multinational corporation.

The resulting dataset includes 7,203 firms. Approximately 8% of them export in a typical

[8] For work incorporating dynamics, see Zheng (2016), who groups choices in clusters such that each choice affects choices in other clusters only through cluster-specific aggregates.

year. While exporting firms often export to a single destination (this being the case for 40% of exporters), 25% of them export to at least four destinations, 10% of them export to at least seven, and 5% of them export to at least ten. By sector, export participation events are concentrated in the manufacturing of other food products (sector 1079) and of plastic products (sector 2220). The most popular destinations are countries that are either geographically close to Costa Rica (e.g., Nicaragua) or relatively large (e.g., the United States). We provide additional descriptive statistics in Appendix B.1.

We complement the firm-level data with data on country characteristics. We collect measures of the geographical distance between countries from CEPII (Mayer and Zignago, 2011), of the languages spoken in each country from Ethnologue (Eberhard et al., 2021), of the content of PTAs from Hofmann et al. (2019), of the tariffs applied to Costa Rican exports from Barari and Kim (2022), and of countries' GDP from the World Bank. Using these data, we build geographical, linguistic, and regulatory distances between countries.

We denote the geographical distance between countries j and j' as $n^g_{jj'}$. As in Head and Mayer (2002), we measure $n^g_{jj'}$ as a population-weighted harmonic mean of distances between cities located in j and j'. Two features of this measure are worth noting. First, it accounts for the location of population within a country; e.g., according to this measure, Russia is closer to Germany (2,290 km) than to China (4,984 km). Second, large countries appear isolated; e.g., while the distance between Switzerland and the UK is 872 km, that between the US and Canada is 1,154 km.

We denote the linguistic distance between countries j and j' as $n^l_{jj'}$. We measure it as the probability two individuals randomly drawn from j and j', respectively, have no shared language. To compute this probability, we use country data on the population shares that speak any given language. Relative to measures based on the commonality of official languages between countries, $n^l_{jj'}$ reflects the prevalence of languages by country, and thus accounts for the fact that languages may be popular in a country without being official; e.g., although the UK and Denmark share no official language, they are linguistically close according to our measure, as a large share of Denmark's population report speaking English.[9]

Our third distance measure between countries j and j' in a year t is an inverse measure of the breadth of the regulatory harmonization imposed by the PTAs of which j and j' are members in t, if any. We denote this measure as $n^a_{jj't}$, refer to it as the regulatory distance between j and j' in t, and build it using the data in Hofmann et al. (2019), which reports whether a PTA contains provisions in each of 52 policy areas. We focus on the seven areas that concern regulatory harmonization, and count in how many of them PTAs

[9]The linguistic distance between the UK and Denmark is 0.11; i.e., the probability that a randomly drawn individual from Denmark does not understand a randomly drawn individual from the UK is 11%.

include a provision.[10] When countries share more than one PTA, we consider only the treaty containing provisions in the largest number of harmonization-focused areas, and compute:

$$n^a_{jj't} = 1 - \frac{1}{7}\left\{\begin{array}{c}\text{number harmonization-focused policy areas in which}\\ \text{the PTA between } j \text{ and } j' \text{ in } t \text{ includes some provision}\end{array}\right\}. \tag{1}$$

This measure is between zero and one. For example, EU countries are bound by an agreement including provisions in all seven harmonization-focused areas and, thus, $n^a_{jj't} = 0$ between them. NAFTA contains provisions in five of the areas; thus, $n^a_{jj't} = 0.29$ between their members. See Appendices B.2 to B.4 for more information on the distances introduced here.

2.2 Correlation in Export Participation Decisions

If geographical, linguistic, or regulatory proximity are sources of cross-country complementarities in firm exports, a firm's export probability in a country j and year t will, all else equal, be larger if it concurrently exports to countries close to j according to any of these three distance measures. To explore whether firm exports in our sample exhibit these correlation patterns, for each firm i, country j, and year t, and for each of the three distance measures we consider, we compute a dummy variable that equals one if firm i exports in year t to at least one country close to j; e.g., for the case of geographical distance, we compute

$$Y^g_{ijt} = \mathbb{1}\Big\{\sum_{j' \neq j} \mathbb{1}\{n^g_{jj'} \leqslant \bar{n}_g\} y_{ij't} > 0\Big\}, \tag{2}$$

where $\mathbb{1}\{\cdot\}$ is an indicator function, $n^g_{jj'}$ is introduced in Section 2, $\bar{n}_g$ is a threshold determining whether we classify two countries as geographically close to each other, and $y_{ij't}$ is a dummy variable that equals one if firm i exports to country j' in year t. We use expressions analogous to that in equation (2) to define two dummy variables, Y^l_{ijt} and Y^a_{ijt}, that equal one if firm i exports in year t to at least one country sufficiently close to j according to the distance measures $n^l_{jj'}$ and $n^a_{jj't}$, respectively. In Appendix B.5, we describe the thresholds we use to classify two countries geographically, linguistically, or regulatory close, and present results for alternative thresholds.

Table 1 presents OLS estimates of regressions of a dummy variable that equals one if firm i exports to a country j in a year t on Y^g_{ijt}, Y^l_{ijt}, and Y^a_{ijt}. Panel A includes estimates of specifications without fixed effects. The results in column (1) show exporting in year t to

[10]These areas cover the harmonization of: sanitary or phytosanitary measures; technical barriers to trade; intellectual property rights; environmental standards; consumer protection laws; statistical methods; competition laws.

Table 1: Conditional Export Probabilities

	Panel A: No Controls				*Panel B: Controlling for Firm-Year Fixed Effects*			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Y^g_{ijt}	0.2622^a			0.2082^a	0.2226^a			0.1957^a
	(0.0092)			(0.0079)	(0.0089)			(0.0081)
Y^l_{ijt}		0.1617^a		0.0752^a		0.1220^a		0.0718^a
		(0.0076)		(0.0054)		(0.0067)		(0.0055)
Y^a_{ijt}			0.0857^a	0.0386^a			0.0517^a	0.0259^a
			(0.0037)	(0.0021)			(0.0026)	(0.0018)
	Panel C: Controlling for Sector-Country-Year Fixed Effects				*Panel D: Controlling for Sector-Country-Year & Firm-Year Fixed Effects*			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Y^g_{ijt}	0.2462^a			0.1955^a	0.2043^a			0.1809^a
	(0.0089)			(0.0076)	(0.0086)			(0.0078)
Y^l_{ijt}		0.1572^a		0.0764^a		0.1160^a		0.0720^a
		(0.0074)		(0.0052)		(0.0066)		(0.0054)
Y^a_{ijt}			0.0809^a	0.0363^a			0.0473^a	0.0207^a
			(0.0035)	(0.0019)			(0.0026)	(0.0018)

Note: ^{a} denotes 1% significance. Standard errors are clustered by firm. The dependent variable is a dummy that equals 1 if i exports to j in t. The covariates are $Y^x_{ijt} = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n^x_{jj'} \leqslant \bar{n}_x\} y_{ij't} > 0\}$ for $x \in \{g, l\}$, and $Y^a_{ijt} = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n^a_{jj't} \leqslant \bar{n}_a\} y_{ij't} > 0\}$, with $\bar{n}_g = 790$ km, $\bar{n}_l = 0.11$ and $\bar{n}_a = 0.43$. In all specifications, the number of observations equals $3,859,618$.

a destination geographically close to a country j increases in 0.26 the probability the firm exports to j in t. The results in columns (2) and (3) indicate this probability increase is 0.16 when the destination is linguistically close to j, and 0.09 when it shares a deep PTA with j. These estimates reveal a strong correlation in firm export choices across countries close to each other, as the average probability a firm exports to a country in a year is below 0.01.

In panels B to D, we present estimates analogous to those in Panel A but for specifications that control for firm-year fixed effects, sector-country-year fixed effects, or both. The point estimates in these panels are only moderately smaller than those in Panel A. The results in Table 1 thus show that firms' export participation decisions in countries geographically or linguistically close to each other, or cosignatories of a deep PTA, are positively correlated, and that factors varying at the firm-year level (e.g., firm productivity, or total number of destinations) or at the sector-country-year level (e.g., market size, or total number of Costa Rican exporters in a destination) are not the main drivers of this correlation.

In Table B.5 in Appendix B.5, we present estimates analogous to those in Table 1 but adding the lagged dependent variable as a control. As in the prior literature, we find there is a strong serial correlation in the firm's export status in a destination, with firms exporting to a country in a year being 0.63 more likely to continue exporting to it the following year. Controlling for the lagged dependent variable decreases the estimates of the coefficients on Y^g_{ijt}, Y^l_{ijt}, and Y^a_{ijt}, but they remain large relative to the mean export probability in a country.

Although consistent with them, the results in Table 1 may not be due to cross-country complementarities in firm exports, as they may be attributable to unobserved firm-country specific export profit shifters that are positively correlated across countries geographically or linguistically close to each other, or that are cosignatories of a deep PTA. Furthermore, it is unclear how one may use the estimates of linear probability models such as those in Table 1 to quantify the contribution of complementarities to overall firm exports. To guide the identification of cross-country complementarities, and to quantify the role these play in determining firm exports, we present below a model that allows both for potential cross-country complementarities and for cross-country correlation in unobserved export determinants.

3 Dynamic Export Model With Complementarities

We present here a partial-equilibrium model in which forward-looking firms choose every period the bundle of countries they export to among a large set of potential destinations. When exporting to a country, firms face variable, fixed, and sunk costs. We allow the fixed costs a firm faces in a destination and period to be smaller if the firm also exports to other countries in the same period. This creates cross-country complementarities: a firm's profits when exporting to multiple countries in a period are weakly larger than the sum of the profits of exporting to each of them individually. Sunk costs make a firm's export choice in a country and period impact export profits in that country in the subsequent period. This creates within-country complementarities: a firm's profits when exporting to a country in two consecutive periods are weakly larger than the sum of the profits of exporting in each of the two periods individually. In the presence of within- and cross-country complementarities, a firm's export choice in a country and period impacts its export profits in other countries and periods. Firms take this into account when choosing where to export. More specifically, firms determine their export bundle in a period after solving an infinite-horizon dynamic combinatorial discrete-choice problem.

We incorporate several shocks that allow export profits to vary flexibly across firms, countries and periods. For faster computation of the solution to the firm's problem, we assume the firm has perfect foresight on some (but not all) of these shocks. We force all shocks on which firms have perfect foresight to stay constant after a terminal period T. The partial perfect foresight assumption can be relaxed at the expense of computation time.

3.1 Setup

Firms produce in country h. Time is discrete. We index periods by $t \geqslant 0$, firms by i, and destinations by j. Firm i is born exogenously at period $\underline{t}_i$ and, once born, is active forever. We denote the first and last sample periods as $\underline{t}$ and $\bar{t}$, respectively, and assume $T > \bar{t}$.

3.2 Marginal Costs, Demand Function, and Market Structure

Firm i has constant marginal production costs w_{it}. Exporting requires incurring in extra variable "iceberg" costs; specifically, firm i must ship τ_{ijt} units of output for a unit to reach country j at period t, and its marginal cost of selling in j at t is thus $\tau_{ijt} w_{it}$.

Conditional on firm i exporting to j at t, the quantity sold q_{ijt}, depends on the price p_{ijt} it sets, the price index P_{jt}, and the market expenditure Y_{jt}, according to the function $q_{ijt} = p_{ijt}^{-\eta} P_{jt}^{\eta-1} Y_{jt}$. Firms set optimal prices in all markets taking as given the market's expenditure and price index and, thus, fix a markup $\eta/(\eta-1)$ over their marginal cost.

3.3 Potential Export Revenues

The assumptions in Section 3.2 imply the potential export revenue of firm i in j at t is

$$r_{ijt} = \left[\frac{\eta}{\eta-1}\frac{\tau_{ijt} w_{it}}{P_{jt}}\right]^{1-\eta} Y_{jt}. \tag{3}$$

We model the impact of variable trade costs on potential export revenues as

$$(\tau_{ijt})^{1-\eta} = \exp(\xi_y y_{ijt-1} + \xi_s + \xi_{jt} + \xi_a \ln(a_{sjt}) + \xi_w \ln(w_{it})), \qquad \text{with} \qquad \xi_y \geqslant 0, \tag{4}$$

where y_{ijt-1} is a dummy variable that equals one if firm i exports to country j at period $t-1$, ξ_s is a term specific to the sector s to which firm i belongs, ξ_{jt} is a country-period term that accounts for trade barriers common to all firms located in country h, a_{sjt} equals one plus the average tariffs country j imposes at t on exports from h in sector s, and, as indicated above, w_{it} denotes marginal production costs. Equations (3) and (4) imply

$$r_{ijt} = \exp(\alpha_s + \alpha_{jt} + \alpha_y y_{ijt-1} + \alpha_a \ln(a_{sjt}) + \alpha_r \ln(r_{iht})), \tag{5}$$

where α_s and α_{jt} are sector and country-period specific terms, respectively, and r_{iht} is firm i's domestic sales at t. The term α_s accounts for the impact of the sector-specific trade cost term ξ_s, and α_{jt} accounts for the impact of the foreign price index P_{jt}, market size Y_{jt}, and variable trade cost component ξ_{jt}. The dependency of r_{ijt} on the export participation dummy y_{ijt-1} accounts for the limited sales firms often obtain upon entering a new market.[11] The term $\alpha_a \ln(a_{sjt})$ accounts for the impact of tariff barriers, and domestic sales r_{iht} proxy for the impact of the firm's marginal production cost, w_{it}. See Appendix C for details.

According to equation (5), potential revenues depend on the lagged export participation dummy y_{ijt-1} and four exogenous terms: the time-invariant term α_s and three time-varying terms comprising the country-period component α_{jt}, log domestic sales $\ln(r_{iht})$, and tariffs a_{sjt}. The time-invariant term and the in-sample values of the time-varying ones are observed or consistently estimated; see sections 2 and 5.1. Out-of-sample, we impose the following restrictions on the distribution of the time-varying exogenous determinants of revenues.[12]

We assume α_{jt} and $\ln(r_{iht})$ are constant after period T and, for all $t \leqslant T$, follow stationary AR(1) processes. Formally, for all j and $t \leqslant T$, we assume $\alpha_{jt} = (X^{\alpha}_{jt})'\beta_\alpha + \rho_\alpha \alpha_{jt-1} + \iota^{\alpha}_{jt}$, with X^{α}_{jt} a vector including a constant, market j's log GDP at $\underline{t}$, and the geographic, linguistic, and regulatory distances between h and j; β_α and ρ_α are parameters with $|\rho_\alpha| < 1$; and, ι^{α}_{jt} is *iid* normally distributed with mean zero and variance σ^2_α. Similarly, for all i and $t \leqslant T$, $\ln(r_{iht}) = (X^r_i)'\beta_r + \rho_r \ln(r_{iht-1}) + \iota^r_{iht}$, with X^r_i a vector including dummies for firm i's sector and location within country h; β_r and ρ_r are parameters with $|\rho_r| < 1$; and ι^r_{it} is *iid* normally distributed with mean zero and variance σ^2_r. Additionally, we assume a_{sjt} is constant out-of-sample; i.e., for all j and s, $a_{sjt} = a_{sj\underline{t}}$ if $t \leqslant \underline{t}$, and $a_{sjt} = a_{sj\bar{t}}$ if $t \geqslant \bar{t}$. Finally, we assume the time series of these three time-varying determinants of revenues are independent of each other and of any other determinant of firm export profits.

3.4 Fixed and Sunk Export Costs

Firms face fixed and sunk costs. Conditional on selling in a market, these costs do not depend on the quantity sold. While fixed costs are paid all periods a firm sells in a country, sunk costs are paid by firms that did not sell in it the prior period. We model fixed costs as:

$$f_{ijt} = g_{jt} - e_{ijt} + \nu_{ijt} + \omega_{ijt}. \tag{6}$$

The first term captures the impact of all distance measures between countries h and j,

$$g_{jt} = \gamma^F_0 + \sum_{x=\{g,l\}} \gamma^F_x n^x_{hj} + \gamma^F_a n^a_{hjt}. \tag{7}$$

[11] These may be due to limited information or customer capital (Fitzgerald et al., 2023) or partial-year effects (Bernard et al., 2017; Gumpert et al., 2020). At the expense of running time when solving the model, we may allow the firm's demand in a country to grow gradually over several years (Ruhl and Willis, 2017).

[12] The need to restrict the out-of-sample distribution of the exogenous determinants of export revenues is due to our model featuring sunk costs and forward-looking firms with rational expectations, which implies firms' export choices in-sample depend on expected potential export revenues out-of-sample; see Section 3.6.

The second term is the main novelty of our framework relative to previous quantitative export dynamics models. It captures cross-country complementarities in export destinations:

$$e_{ijt} = \sum_{j' \neq j} y_{ij't} c_{jj't}, \tag{8}$$

where the fixed cost reduction in country j for a firm exporting to country j' is

$$c_{jj't} = \sum_{x=\{g,l\}} \gamma_x^E (1 + \varphi_x^E n_{hj}^x) \exp(-\kappa_x^E n_{jj'}^x) + \gamma_a^E (1 + \varphi_a^E n_{hjt}^a) \exp(-\kappa_a^E n_{jj't}^a), \tag{9}$$

with $(\gamma_x^E, \varphi_x^E) \geqslant 0$ for $x = \{g, l, a\}$. For all three distance measures we consider, equation (9) allows the fixed cost reduction a firm enjoys in a country j if it also exports to a country j' to depend on the distance between j and j' and between j and the firm's home market h. For example, for the case of linguistic distances, a firm exporting to j' enjoys a fixed cost reduction in j equal to the product of a constant γ_l^E, a function $1 + \varphi_l^E n_{hj}^l$ of the distance between countries h and j, and a function $\exp(-\kappa_l^E n_{jj'}^l)$ of the distance between j and j'.[13]

Imposing $(\gamma_x^E, \varphi_x^E) \geqslant 0$ for $x = \{g, l, a\}$ implies $c_{jj't} \geqslant 0$ for all (j, j', t), ruling out the possibility that adding an export destination increases fixed costs in other countries. Along with the rest of the model, this sign restriction on $c_{jj't}$ implies the firm's export participation decisions are not substitutable. This restriction is necessary for our algorithm to correctly solve the optimization problem determining firms' export bundles; see Section 4.1.

The third term in fixed export costs, ν_{ijt}, is a an unobserved (to the researcher) firm-country-period variable whose distribution in all periods prior to terminal period T is independent of all other determinants of export profits, and satisfies the following restrictions:

$$\nu_{ijt} \sim \mathbb{N}(0, \sigma_\nu^2), \qquad \text{for all } i,\ j, \text{ and } t, \tag{10a}$$

$$\nu_{ijt} \perp\!\!\!\perp \nu_{i'j't'}, \qquad \text{if } i \neq i' \text{ or } t \neq t', \tag{10b}$$

$$\rho_{jj't} = \sum_{x=\{g,l\}} \gamma_x^N \exp(\kappa_x^N n_{jj'}^x) + \gamma_a^N \exp(\kappa_a^N n_{jj't}^a), \qquad \text{if } j \neq j', \tag{10c}$$

where $\rho_{jj't}$ is the correlation coefficient between ν_{ijt} and $\nu_{ij't}$. In our estimation, we impose restrictions on (γ_x^N, κ_x^N) for $x = \{g, l, a\}$ that guarantee that the resulting correlation matrix is valid. By allowing for ν_{ijt} to be potentially correlated across countries, we allow for the correlation patterns in firm exports shown in Table 1 to be due not to complementarities but

[13] By allowing the complementarities between j and j' to depend on j's distance to country h –which corresponds to Costa Rica in our setting– we let, e.g., the complementarities between German-speaking countries differ from the complementarities between countries in which Spanish is the dominant language.

to correlated unobserved determinants of export profits. From T onwards, ν_{ijt} is constant.[14]

The fourth term, ω_{ijt}, is an unobserved (to the researcher) variable that is independent of all other determinants of profits and has two points of support, $\underline{\omega}$ and $\overline{\omega}$. More specifically,

$$\omega_{ijt} \perp\!\!\!\perp \omega_{i'j't'} \qquad \text{if } i \neq i',\ j \neq j' \text{ or } t \neq t', \tag{11a}$$

$$P(\omega_{ijt} = \omega) = \begin{cases} p & \text{if } \omega = \underline{\omega}, \\ 1-p & \text{if } \omega = \overline{\omega}. \end{cases} \tag{11b}$$

To simplify the model estimation, we set $(\underline{\omega}, \overline{\omega}) = (0, \infty)$, thus modeling ω_{ijt} as a "blocking" shock that prevents firm i from exporting to country j at t. Equation (11) specifies the distribution of $\omega_{it} \equiv \{\omega_{ijt}\}_{j=1}^{J}$ in all periods; thus, ω_{it} may vary over time even after T.[15]

We model sunk costs more parsimoniously than fixed costs. Specifically, sunk costs in a market j and period t are only a function of the distance between countries h and j:

$$s_{jt} = \gamma_0^S + \sum_{x=\{g,l\}} \gamma_x^S n_{hj}^x + \gamma_a^S n_{hjt}^a, \qquad \text{with} \qquad s_{jt} \geqslant 0, \text{ for all } (j,t). \tag{12}$$

Sunk costs allow for within-country complementarities in firm export decisions.[16]

3.5 Static Export Profits

The assumptions in Section 3.2 imply potential export revenues net of variable trade costs equal $\eta^{-1} r_{ijt}$. Netting out also fixed and sunk export costs, and using the expressions in equations (6) and (8), we can write the potential export profits of firm i in j at t as

$$\pi_{ijt}(y_{it}, y_{ijt-1}, \omega_{ijt}) = u_{ijt}(y_{ijt-1}, \omega_{ijt}) + \sum_{j' \neq j} y_{ij't} c_{jj't}, \tag{13}$$

with

$$u_{ijt}(y_{ijt-1}, \omega_{ijt}) = \eta^{-1} r_{ijt} - (g_{jt} + \nu_{ijt} + \omega_{ijt}) - (1 - y_{ijt-1}) s_{jt}. \tag{14}$$

[14] Equation (10b) imposes ν_{ijt} is serially uncorrelated for all $t < T$. All persistence in export status is thus attributed in our model to persistent observed shifters of export profits or to sunk export costs. We consider in Appendix F.8 an alternative model in which we assume ν_{ijt} is permanent; i.e., $\nu_{ijt} = \nu_{ij}$ for all t.

[15] In our model, firm i will not export to country j at period t if $\omega_{ijt} = \infty$. Thus, we may interpret ω_{ijt} as a shock to the firm's consideration set, accounting thus for the possibility that firms do not consider exporting to some foreign countries, and that their consideration set varies for reasons unknown to the researcher.

[16] Although our model does not feature cross-country complementarities in sunk costs, the introduction of sunk costs and cross-country complementarities in fixed costs implies that expected shocks to export profits in a country j at a period t may impact the firm's export choice in some other country j' at a period $t' < t$. Thus, our model is able to reproduce some of the predictions of models with cross-country complementarities in sunk export costs.

The variable r_{ijt} is defined in equation (5), and $y_{it} \equiv \{y_{ijt}\}_{j=1}^{J}$ identifies the bundle of export destinations of firm i at period t. Total export profits of firm i at period t thus are

$$\pi_{it}(y_{it}, y_{it-1}, \omega_{it}) = \sum_{j=1}^{J} y_{ijt}\pi_{ijt}(y_{it}, y_{ijt-1}, \omega_{ijt}). \tag{15}$$

3.6 Optimal Export Choice

The firm chooses every period a set of export destinations maximizing its expected discounted sum of current and future profits. At any t, we assume firm i knows the distance measures between all countries, the true value of all model parameters, and the information set

$$\mathcal{J}_{it} = (\{x_{it'}\}_{t' \geqslant t}, y_{it-1}, \omega_{it}) \qquad \text{with} \qquad x_{it'} = (\nu_{it'}, \alpha_{t'}, a_{st'}, r_{iht'}), \tag{16}$$

where, for any t', $\nu_{it'} = \{\nu_{ijt'}\}_{j=1}^{J}$, $\alpha_{t'} = \{\alpha_{jt'}\}_{j=1}^{J}$, and $a_{st'} = \{a_{sjt'}\}_{j=1}^{J}$. Therefore, $x_{it'}$ includes all period-t' realized export profit shocks known to firm i at any period $t \leqslant t'$. Every firm i thus knows at any t the value of all exogenous determinants of current and future potential export profits except for the future fixed costs shocks $\{\omega_{it'}\}_{t'>t}$.

The problem firm i solves when choosing its period t export bundle may be written as

$$V_{it}(y_{it-1}, \omega_{it}) = \max_{y_{it} \in \{0,1\}^J} \mathbb{E}_{it}\Big[\pi_{it}(y_{it}, y_{it-1}, \omega_{it}) + \delta V_{it+1}(y_{it}, \omega_{it+1})\Big], \tag{17}$$

where $\mathbb{E}_{it}[\cdot]$ denotes the expectation with respect to the data generating process conditional on $\mathcal{J}_{it}$ (i.e., expectations are rational); the function $V_{it}(\cdot)$, firm i's value function at t, implicitly conditions on the path of shocks $\{x_{it'}\}_{t' \geqslant t}$ (i.e., $V_{it}(y_{it-1}, \omega_{it}) = V(\{x_{it'}\}_{t' \geqslant t}, y_{it-1}, \omega_{it})$); and $\delta < 1$ is the discount factor. Given equations (13), (16), and (17), we can rewrite

$$V_{it}(y_{it-1}, \omega_{it}) =$$
$$\max_{y_{it} \in \{0,1\}^J} \Big\{ \sum_{j=1}^{J} y_{ijt}(u_{ijt}(y_{ijt-1}, \omega_{ijt}) + \sum_{j' \neq j} y_{ij't}c_{jj't}) + \delta \mathbb{E}_{it} V_{it+1}(y_{it}, \omega_{it+1}) \Big\}. \tag{18}$$

For all values of $\{x_{it'}\}_{t' \geqslant t}$, $V_{it}(\cdot)$ is bounded, and a solution to the problem in equation (18) thus exists; see Appendix E.2.2. We denote firm i's optimal policy function at t as

$$o_{it}(y_{it-1}, \omega_{it}) = (o_{i1t}(y_{it-1}, \omega_{it}), \ldots, o_{iJt}(y_{it-1}, \omega_{it})) \tag{19}$$

where $o_{ijt}(\cdot)$ is a function that equals one if firm i exports to country j at t, and zero otherwise. As x_{it} is constant from period T onwards (see sections 3.3 and 3.4), it holds that $V_{it}(\cdot) = V_{iT}(\cdot)$ for all $t \geqslant T$ and, consequently, $o_{it}(\cdot) = o_{iT}(\cdot)$ for all $t \geqslant T$. The firm's problem is thus non-stationary until terminal period T, and stationary thereafter.

4 Solution Algorithm

We introduce an algorithm to solve the problem in equation (18). We discuss the algorithm's formal properties in Appendix A. In Appendix D.2, we use a simple setting to illustrate how the algorithm works. Given a period t and a sequence of shocks $\{x_{it'}\}_{t' \geqslant t}$, the firm's optimization problem in equation (18) has three properties that make solving for the value of the policy function $o_{it}(y_{it-1}, \omega_{it})$ at every state (y_{it-1}, ω_{it}) computationally challenging:

P.1 *Large discrete choice set.* The choice set $\{0,1\}^J$ is discrete and has cardinality 2^J.

P.2 *Integration over a discrete variable with many points of support.* For any choice y_{it}, evaluating the firm's objective function requires integrating numerically next period's value function, $V_{it+1}(y_{it}, \omega_{it+1})$, over ω_{it+1}, whose support includes 2^J points.

P.3 *Large state space.* As y_{it-1} and ω_{it} are of dimension 2^J, the state space has 2^{2J} points.

These properties imply the choice set, the support of the random variable one must integrate over, and the state space grow exponentially in J. Allowing firms to export to a reasonable set of countries thus makes their optimization problem computationally challenging to solve with standard algorithms for solving dynamic problems. Specifically, as the firm's problem is non-stationary for all $t \leqslant T$, property P.3 implies one must solve $2^{2J}(T - \underline{t}_i + 1)$ optimization problems to compute firm i's export choices in all periods in which it is active and in all points in the state space. Properties P.1 and P.2 make finding the solution to each of these problems computationally challenging.

To overcome the challenges posed by properties P1 to P3, we develop a new solution algorithm. Our algorithm does not yield the value of the policy function $o_{it}(y_{it-1}, \omega_{it})$ for every feasible state (y_{it-1}, ω_{it}). Instead, we consider each firm i independently and, given a sequence of shocks $\{x_{it}\}_{t \geqslant \underline{t}_i}$, we compute for each $t \geqslant \underline{t}_i$ the value of $o_{it}(y_{it-1}, \omega_{it})$ at one state, which we denote as $(\check{y}_{it-1}, \check{\omega}_{it})$. The state $(\check{y}_{it-1}, \check{\omega}_{it})$ is reached when the shocks $\{\omega_{it}\}_{t'=\underline{t}_i}^{t}$ follow a particular path $\{\check{\omega}_{it'}\}_{t'=\underline{t}_i}^{t}$ and the firm makes the optimal choices at all periods prior to t. More specifically, $\check{y}_{it-1}$ is determined by the following procedure:

$$\check{y}_{it'} = o_{it'}(\check{y}_{it'-1}, \check{\omega}_{it'}), \qquad \text{for } t' = \underline{t}_i, \ldots, t-1, \text{ with initial value } \check{y}_{i\underline{t}_i - 1} = 0_J, \tag{20}$$

with 0_J a $J \times 1$ vector of zeros. Thus, according to this procedure, $\check{y}_{i\underline{t}_i} = o_{i\underline{t}_i}(0_J, \check{\omega}_{i\underline{t}_i})$, $\check{y}_{i\underline{t}_i+1} = o_{i\underline{t}_i+1}(\check{y}_{i\underline{t}_i}, \check{\omega}_{i\underline{t}_i+1})$, and so on. In practice, the sequences of shocks $\{x_{it'}\}_{t' \geqslant \underline{t}_i}$ and $\{\check{\omega}_{it'}\}_{t' \geqslant \underline{t}_i}$

defining the states at which we solve for firm i's optimal choices correspond to either the sequences of shocks observed in the data (or fixed to counterfactual values) or, when the shock is unobserved, to random sequences drawn from their distribution.

As our model is dynamic and firms are forward-looking, solving the optimization problem of firm i at period t and state $(\check{y}_{it-1}, \check{\omega}_{it})$ requires *some knowledge* of how the firm will subsequently behave at any state that may be reached from $(\check{y}_{it-1}, \check{\omega}_{it})$. However, it may not require knowing *exactly* the firm's optimal export bundle in all states that may subsequently be reached; e.g., if firm i's potential export profits in a country j at period t and state $(\check{y}_{it-1}, \check{\omega}_{it})$ are sufficiently high, its optimal decision may be to export to j at this state regardless of its optimal decision in subsequent periods. Our algorithm uses this idea and computes the optimal choice of a firm i at a period t and state $(\check{y}_{it}, \check{\omega}_{it})$ using *bounds* on the firm's optimal choice at future states.

Our algorithm has several steps. In each one, we obtain upper and lower bounds on the solution to the firm's problem at the states of interest. If the bounds coincide, they equal the solution as well. If they do not, we move to the next step.

Step 1. For a country j and period t, assume we know for all $j' \neq j$ and $t' \geqslant t$ a value $\bar{b}_{ij't'}$ such that $\bar{b}_{ij't'} \geqslant o_{ij't'}(y_{it'-1}, \omega_{it'})$ for all $(y_{it'-1}, \omega_{it'})$. We can then solve the firm's problem in j at t while conditioning on the upper bound $\bar{b}_{ij't'}$ for each $j' \neq j$ and $t' \geqslant t$:

$$\bar{V}_{ijt}(y_{ijt-1}, \omega_{ijt}) = \max_{y_{ijt} \in \{0,1\}} \Big\{ y_{ijt}(u_{ijt}(y_{ijt-1}, \omega_{ijt}) + \sum_{j' \neq j} \bar{b}_{ij't}(c_{jj't} + c_{j'jt})) + \delta \mathbb{E}_{it} \bar{V}_{ijt+1}\left(y_{ijt}, \omega_{ijt+1}\right) \Big\}. \tag{21}$$

The complementarities in our model imply that the solution to this problem is an upper bound on the firm's optimal choice in j at t; i.e., the solution is a function $\bar{o}_{ijt}(\cdot)$ such that $\bar{o}_{ijt}(y_{ijt-1}, \omega_{ijt}) \geqslant o_{ijt}(y_{ijt-1}, \omega_{ijt})$ for all $(y_{ijt-1}, \omega_{ijt})$. Importantly, the problem in equation (21) does not have any of the three properties P.1 to P.3 described above: the choice variable is binary, one only needs to integrate over the binary variable ω_{ijt+1}, and the vector $(y_{ijt-1}, \omega_{ijt})$ only takes four values.[17]

Given upper bounds $\bar{b}_{it} = (\bar{b}_{i1t}, \ldots, \bar{b}_{iJt})$ for all $t \geqslant \underline{t}_i$, we may solve the problem in equation (21) for all countries and periods, obtaining in this way upper-bound policy functions

$$\bar{o}_{it}(y_{it-1}, \omega_{it}) = (\bar{o}_{i1t}(y_{i1t-1}, \omega_{i1t}), \ldots, \bar{o}_{iJt}(y_{iJt-1}, \omega_{iJt})), \qquad \text{for all } \underline{t}_i \leqslant t \leqslant T. \tag{22}$$

More specifically, we use value function iteration to solve for period-T value and policy

[17] These are $(0, \underline{\omega})$, $(0, \overline{\omega})$, $(1, \underline{\omega})$, and $(1, \overline{\omega})$. As $\overline{\omega} = \infty$ in our application, $o_{ijt}(0, \overline{\omega}) = o_{ijt}(1, \overline{\omega}) = 0$ for all i, j and t, and we only need to compute $o_{ijt}(0, \underline{\omega})$ and $o_{ijt}(1, \underline{\omega})$.

functions $\bar{V}_{ijT}(\cdot)$ and $\bar{o}_{ijT}(\cdot)$, and backward induction to solve for $\{\bar{V}_{ijt}(\cdot)\}_{t=\underline{t}_i}^{T-1}$ and $\{\bar{o}_{ijt}(\cdot)\}_{t=\underline{t}_i}^{T-1}$.

The upper-bound policies $\{\bar{o}_{it}(\cdot)\}_{t\geqslant\underline{t}_i}$ we obtain depend on the constant upper bounds $\{\bar{b}_{it}\}_{t\geqslant\underline{t}_i}$ we use: the tighter these are, the tighter the resulting upper-bound policies will be. To initialize our algorithm, we use constant upper bounds implying the firm exports in all countries and periods. We denote these with a zero superscript (i.e., $\bar{b}_{ijt}^{[0]} = 1$ for all j and t) and use them to solve the problem in equation (21) for every country and period, obtaining in this way upper-bound policies $\bar{o}_{it}^{[0]}(\cdot)$ for all $t \geqslant \underline{t}_i$. Using these policies, we compute new constant upper bounds, which we use to solve again the problem in equation (21) and obtain new upper-bound policies. Generally, we implement an iterative algorithm computing each iteration's constant upper bounds using the policies obtained in the prior iteration. More specifically, to compute the period-t iteration-$(n+1)$ constant upper bound, we evaluate the period-t iteration-n upper-bound policy at the feasible state at which the firm is most likely to export at t. This is the state reached when, for all $t' \leqslant t$, the vector of shocks $\omega_{it'}$ equals the smallest value in its support and the firm chooses the bundle prescribed by $\bar{o}_{it'}^{[n]}(\cdot)$. That is, for a firm i, we compute the period-t iteration-$(n+1)$ constant upper bound as:

$$\bar{b}_{it'}^{[n+1]} = \bar{o}_{it'}^{[n]}(\bar{b}_{it'-1}^{[n+1]}, \underline{\omega}_J), \qquad \text{for } t' = \underline{t}_i, \ldots, t, \text{ with initial value } \bar{b}_{i\underline{t}_i-1}^{[n+1]} = 0_J, \tag{23}$$

As shown in Appendix A, these bounds get tighter with every iteration and converge in a finite number of iterations.

We denote the converged upper-bound policies as $\{\bar{o}_{it}^{*}(\cdot)\}_{t\geqslant\underline{t}_i}$, obtain lower-bound policies $\{\underline{o}_{it}^{*}(\cdot)\}_{t\geqslant\underline{t}_i}$ in a similar way, and use both to obtain bounds on the firm choices at the states of interest. Formally, denoting the upper and lower bounds at t at the state $(\check{y}_{it-1}, \check{\omega}_{it})$ as $\bar{\check{y}}_{it}$ and $\underline{\check{y}}_{it}$, respectively, we compute $\bar{\check{y}}_{it}$ through the following iterative procedure:

$$\bar{\check{y}}_{it'} = \bar{o}_{it'}^{*}(\bar{\check{y}}_{it'-1}, \check{\omega}_{it'}), \qquad \text{for } t' = \underline{t}_i, \ldots, t, \text{ with initial value } \bar{\check{y}}_{i\underline{t}_i-1} = 0_J, \tag{24}$$

and compute $\underline{\check{y}}_{it}$ analogously. If $\bar{\check{y}}_{it} = \underline{\check{y}}_{it}$ for all $t \geqslant \underline{t}_i$, these bounds identify the firm's optimal choices along the path of interest. If they differ for some $t \geqslant \underline{t}_i$, we move to step 2.

Step 2-5. In steps 2 to 5, we improve on the bounds $\bar{\check{y}}_{it}$ and $\underline{\check{y}}_{it}$ at the periods at which these differ. For example, for a period t at which $\bar{\check{y}}_{it} > \underline{\check{y}}_{it}$, we obtain weakly smaller upper bounds $\bar{\check{y}}_{it}$ by solving problems that, compared to that in equation (21), are computationally harder to solve but that yield weakly smaller upper-bound policy functions $\bar{o}_{it}(\cdot)$. We describe here succinctly these alternative problems, and provide more details in Appendix D.1.

In step 2, to compute a tighter upper-bound policy $\bar{o}_{it}(\cdot)$, we solve a problem analogous to that in equation (21) but condition on the period-t state of interest $(\check{y}_{it}, \check{\omega}_{it})$ when building the constant upper bounds on the firm's optimal choices in subsequent periods. We hence use constant upper bounds $\{\bar{b}_{it'}\}_{t' \geqslant t}$ that are weakly lower than those computed in equation (23). In steps 3 and 4, instead of solving for the firm's optimal policy at a country j and period t conditioning on constant upper bounds in all other countries and subsequent periods, we condition on contingent upper-bound functions $\bar{b}_{ij't'}(y_{ij't'-1}, \omega_{ij't'})$ for a subset of countries $j' \neq j$ and periods $t' \geqslant t$. Solving the resulting optimization problem requires integrating over the corresponding variables $\omega_{ij't'}$ and, thus, is computationally more costly than solving the problem in equation (21). Finally, in step 5, we no longer solve for the optimal export path one country at a time, but instead solve for the optimal path in subsets of countries. The resulting upper bounds are thus tighter, as they internalize the complementarities among the countries whose policies we simultaneously solve for.

4.1 Discussion

Two model features are necessary for our algorithm to provide valid and computationally feasible bounds on the firm's optimal choices. First, the function the firm maximizes when making choices at any period and state is supermodular; i.e., the objective function in the optimization problem in equation (18) is supermodular. Supermodularity of the objective function implies we can compute upper and lower bounds on the firm's optimal policy function by iteratively solving for the firm's optimal policy in a subset of countries and periods while conditioning on upper and lower bounds, respectively, on the firm's optimal choices in all other countries and subsequent periods. In our model, the objective function is supermodular because of possible complementarities in export choices across countries within a period (due to fixed costs being weakly smaller when firms concurrently export to several destinations) and across periods within a country (due to weakly positive sunk costs). The specific source of complementarities is irrelevant for the validity of the solution algorithm.

Second, given bounds on the firm's optimal choices in all other countries, the firm's dynamic optimization problem for one country (or a small set of them) is computationally tractable. For this, the dimensionality of the state vector in the country-specific problem in equation (21) must be small. In our model, this vector takes only four values, as $y_{ijt-1} \in \{0, 1\}$ and $\omega_{ijt} \in \{\underline{\omega}, \bar{\omega}\}$ for all i, j, and t. As long as the state space of the country-specific problem is small, our solution algorithm is however still feasible if, e.g., ω_{ijt} has a distribution with more than two points of support; per-period profits in a country j depend on multiple lags of the firm's export participation choice in j; or, the firm's information is more limited than assumed in equation (16).

As discussed in Appendix D.3, the share of export choices solved in each step of the algorithm, and the associated computing time, depend on the model parameter values.

When these equal the baseline estimates (see Section 6), step 1 of the algorithm runs in close to two minutes and yields the solution to 99.72% of the more than 22 million choices we solve for when computing the model's sample predictions, and all five steps of the algorithm together find in less than 13 minutes the solution to 99.89% of all choices.[18] The unsolved choices are concentrated in countries sharing complementarities with a large number of other destinations; i.e., according to our estimates, those sharing deep PTAs with many other countries. At each step of the algorithm, the share of choices solved increases, and the computing time decreases, as the gravity component in fixed or sunk costs gets larger (i.e., as the value of the parameters entering equations (7) or (12) increase) and as complementarities get smaller (i.e., as γ_x^E or φ_x^E decrease, or as κ_x^E increases, for $x = \{g, l, a\}$).

5 Estimation Procedure

We estimate the model in two steps. First, we estimate the demand elasticity and time series process of potential export revenues. Second, we estimate fixed and sunk costs.

5.1 First Step

We assume $r_{ijt}^{obs} = (r_{ijt} + \epsilon_{ijt})y_{ijt}$, where r_{ijt}^{obs} denotes observed export revenues, ϵ_{ijt} captures measurement error and, as a reminder, r_{ijt} is the potential export revenue of firm i in country j at t, and y_{ijt} is a dummy variable that equals one if i exports to j at t. Using d_s and d_{jt} to denote vectors of sector and country-year dummies, respectively, we assume $\mathbb{E}[\epsilon_{ijt}|y_{ijt-1}, d_s, d_{jt}, a_{sjt}, r_{iht}, y_{ijt} = 1] = 0$ and use a Poisson pseudo-maximum likelihood estimator and data on the sample of firms, countries, and years for which $y_{ijt} = 1$ to obtain estimates of the parameters entering equation (5); i.e., $(\alpha_y, \alpha_a, \alpha_r, \{\alpha_{jt}\}_{jt}, \{\alpha_s\}_s)$.[19]

We also assume $r_{it}^{obs} = r_{it} + \varepsilon_{it}$, where r_{it}^{obs} denotes the observed total sales of firm i in year t, r_{it} is this variable's true value, and ε_{it} accounts for measurement error. As firms are monopolistically competitive and face in all markets a demand function with constant elasticity equal to η, it holds that $r_{it} = (\eta/(\eta - 1))vc_{it}$, where vc_{it} is the total variable costs of firm i in year t, which we measure as the sum of the wage bill and total expenditure in materials. Assuming $\mathbb{E}[\varepsilon_{it}|vc_{it}] = 0$, we use a non-linear least squares estimator to obtain a consistent estimate of η.

[18] Times measured at Princeton University's Della cluster using 44 processors with 20 GB of memory each.

[19] Our procedure is compatible with interpreting ϵ_{ijt} as a revenue term unknown to firms when choosing where to export. Assuming instead firms make this choice on the basis of such unobserved terms would force us (for computational reasons) to limit the number of parameters entering revenues; e.g., we may need to substitute the fixed effects $\{\alpha_{jt}\}_{jt}$ and $\{\alpha_s\}_s$ by functions of observed covariates and few parameters.

Finally, given estimates of α_{jt} for all sample countries and years, and data on domestic sales for all sample firms and years, we compute OLS estimates of the parameters of the first-order autoregressive processes for α_{jt} and $\ln(r_{iht})$; see Section 3.3.

5.2 Second Step

Given first-step estimates, we use a Simulated Method of Moments (SMM) estimator to obtain estimates of the fixed and sunk cost parameters; see equations (7) to (12). In Section 5.2.1, we use a simple example to illustrate the approach we follow to separately identify the parameters that, according to equation (9), determine the strength of cross-country complementarities in fixed costs from those that, according to equation (10c), determine the strength of the cross-country correlation in unobserved fixed cost determinants. In Section 5.2.2, we describe our SMM estimator.

5.2.1 Identification of Cross-Country Export Complementarities

Consider a setting with three destinations $j = \{1, 2, 3\}$. Countries 1 and 2 are identical except in their connection to country 3, which is "connected" to country 2 but not to country 1. Complementarities and the correlation in the fixed cost term ν_{ijt} thus equal zero between all country pairs except possibly between countries 2 and 3; i.e., $c_{jj't} = \rho_{jj't} = 0$ if $j = 1$ or $j' = 1$, $c_{23t} = c_{32t} = \bar{c}$, and $\rho_{23t} = \rho_{32t} = \bar{\rho}$ for $(\bar{c}, \bar{\rho}) \geqslant 0$. See Appendix F.1 for details.

To focus on the identification of $\bar{c}$ and $\bar{\rho}$, consider a researcher that knows the value of all other parameters and, besides the variables described in Section 2, observes potential export revenues for all firms, countries, and periods. Then, $\bar{c}$ and $\bar{\rho}$ are identified by the moments

$$\mathbb{m}_1 = \mathbb{E}[y_{i2t} - y_{i1t}] \qquad \text{and} \qquad \mathbb{m}_2 = \mathbb{C}[y_{i2t}, y_{i3t}], \tag{25}$$

where, generally, $\mathbb{m}_1$ captures the difference in export probabilities across destinations that differ only in the size of the countries "connected" to them (in our setting, country 2 is connected to country 3 while country 1 is not; countries 1 and 2 are otherwise identical), and $\mathbb{m}_2$ captures the covariance in firm choices across "connected" countries (countries 2 and 3). As Table F.1 in Appendix F.1 shows, both moments equal zero when there are no complementarities and ν_{ijt} is independent across countries; i.e., when $\bar{c} = \bar{\rho} = 0$. Correlation in unobservables in the absence of complementarities (i.e., $\bar{\rho} > 0$ and $\bar{c} = 0$) yields correlated export choices without affecting the difference in export probabilities between connected and isolated countries (i.e., $\mathbb{m}_2 > 0$ and $\mathbb{m}_1 = 0$). Complementarities alone (i.e., $\bar{c} > 0$ with $\bar{\rho} = 0$) make both moments positive. This suggests an identification strategy in which $\mathbb{m}_1$

identifies the strength of the complementarities and, given these, $\mathbb{m}_2$ identifies the correlation in unobserved determinants of export profits. This logic is however incorrect, as $\mathbb{m}_1$ is also affected by the correlation in unobserved determinants of profits whenever complementarities are non-zero; i.e., $\mathbb{m}_1$ is also affected by $\bar{\rho}$ when $\bar{c} > 0$. What is true is that $\mathbb{m}_1$ and $\mathbb{m}_2$ are differentially affected by $\bar{c}$ and $\bar{\rho}$, and jointly identify them; see Figure F.1 in Appendix F.1. To estimate our model, we use moments analogous to $\mathbb{m}_1$ and $\mathbb{m}_2$, but adjusted to account for the fact that no two countries in the data are identical in every dimension except the size of their "connected" countries, and that the degree in which any two countries are connected in our model (i.e., their proximity measures) is continuous.

5.2.2 Details on SMM Estimator

Consider a vector z_i that includes all first-step estimates (see Section 5.1) and all observed to the researcher firm i's payoff-relevant variables. That is, besides the first-step estimates, z_i includes, for all sample years, firm i's domestic sales and exports by destination, tariffs by destination for i's sector, and, for all country pairs, the distance measures introduced in Section 2. Consider also a vector χ_i including all firm i's payoff-relevant variables unobserved to the researcher: the vectors of fixed cost shocks ν_{it} and ω_{it} for all years, and, for non-sample years, the vector of foreign countries' export revenue shifters α_t and firms' domestic sales. Finally, consider vectors y_i^{obs} and $y_i^s(\theta)$ of observed and model-implied, respectively, export choices in all countries and sample years. Specifically, $y_i^s(\theta)$ includes the model-implied choices given z_i, a vector θ of values for all parameters estimated in the second step, and a draw χ_i^s from the distribution of χ_i conditional on z_i. We can then write each of the $k = 1, \ldots, K$ moments we use in our SMM estimator as

$$\frac{1}{M}\sum_{i=1}^{M}\left\{m_k(y_i^{obs}, z_i, x) - \frac{1}{S}\sum_{i=1}^{S} m_k(y_i^s(\theta), z_i, x)\right\} = 0, \tag{26}$$

where M is the number of sample firms, $m_k(\cdot)$ is k's moment function, and x is a vector of export potentials for every sector, destination, and sample year. We estimate these export potentials as the corresponding importer fixed effect in a gravity equation estimated using sectoral trade data between all countries other than Costa Rica. In Appendix F.2, we summarize the distribution of export potentials and show that, controlling for the export potential of a destination, firms are more likely to export to destinations whose (geographical, linguistic or regulatory) neighbors' export potential is larger.

We use 89 moments. For expositional purposes, we organize them in three blocks. In a first block, with the goal of identifying the parameters that determine the level of fixed and sunk costs and how these vary with the distance between the firm's home country and each destination (i.e., γ_0^F, γ_0^S, and $\{(\gamma_x^F, \gamma_x^S)\}_{x=\{g,l,a\}}$), we use as moments the firm's export participation and survival probabilities by groups of destinations that differ in their distances to the firm's home country. In a second block, to identify the parameters determining the strength of export complementarities (i.e., $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$), we use moments that, similar to $\mathbb{m}_1$ in equation (25), capture firm export probabilities by groups of destinations that are similar in their size and distances to the firm's home country but different in the export potential of the countries close to them geographically or linguistically, or that share with them a deep PTA. Finally, to identify the parameters of the distribution of the unobserved terms ν_{it} and ω_{it} (i.e., σ_ν, p, and $\{(\gamma_x^N, \kappa_x^N)\}_{x=\{g,l,a\}}$), we combine moments that, similar to $\mathbb{m}_2$ in equation (25), capture the cross-country correlation in firm choices, with moments that capture the correlation in exports across firms, and moments that capture the frequency with which we observe short-lived changes in a firm's export status in a destination.

We include in Appendix F.3 the full list of moments. We provide details on our SMM estimator in Appendix F.4. In Appendix F.5, we explore the robustness of our estimates to alternative realizations of the simulation draws χ_i^s we use in our moments.

6 Estimation Results

We summarize here our parameter estimates. We discuss additional details in Appendix F.6.

6.1 First-step Estimates: Potential Export Revenue Parameters

We estimate the export revenue parameters using the 13,293 firm-country-year observations with positive exports. The estimate of α_y is 1.86 (robust s.e. equal to 0.07), implying firm potential export revenues grow significantly between the first and second year of exports to a destination. The estimate of α_a, which equals the elasticity of potential export revenues to tariffs, is -3.83 (s.e. equal to 0.07). If trade costs moved one-to-one with tariffs, this estimate would imply a demand elasticity η equal to 4.83. When estimating η as described in Section 5.1 (i.e., using information on total revenues and variable costs for all 44,785 firm-year sample observations), we obtain an estimate of 5.71 (s.e. equal to 0.49). Following Das et al. (2007), we adopt this latter estimate as our baseline. The estimate of α_r, the elasticity of potential export revenues to domestic sales, is 0.29 (s.e. equal to 0.04), reflecting a positive cross-firm correlation in sales between the domestic and foreign markets.

In Figure F.7, we summarize the estimates of the country-year fixed effects in export revenues: countries with large estimated values of α_{jt} tend to be geographically close to

Costa Rica (e.g., Guatemala) or large (e.g., the US), and countries with small values tend to be geographically far from Costa Rica (e.g., Russia) or small (e.g., Oman). When using the 467 estimated values $\{\hat{\alpha}_{jt}\}_{jt}$ to estimate the parameters of the stochastic process of α_{jt} (see Section 3.3), we obtain an estimate of its autocorrelation parameter ρ_α equal to 0.69 (s.e. clustered by destination equal to 0.06), an estimate of the standard deviation σ_α of its innovations equal to 0.63, and estimates implying the mean of α_{jt} increases in country j's GDP and geographical proximity to Costa Rica. Similarly, when estimating the parameters of the autoregressive process for the firm's log domestic sales (see Section 3.3), we obtain an estimate of its autocorrelation parameter ρ_r equal to 0.86 (s.e. clustered by firm equal to 0.01), and an estimate of the standard deviation σ_r of its innovations equal to 0.87.

6.2 Second-step Estimates: Fixed and Sunk Costs Parameters

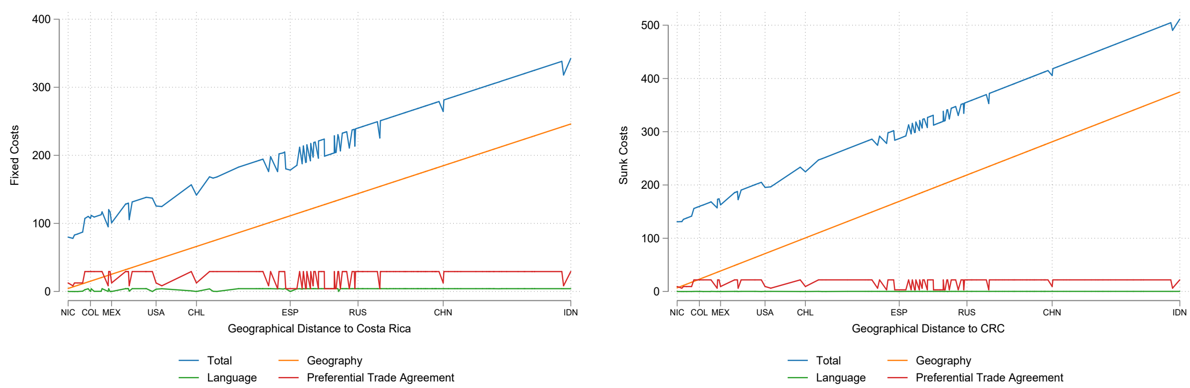
As shown in Figure 1, the estimates of the fixed and sunk cost parameters (see Table F.4) imply the gravity component of fixed costs (see equation (7)), and sunk costs, are well approximated by a constant (which equals \$63,000 in the case of fixed costs and \$115,000 in the case of sunk costs) plus a term that increases in the geographical distance between the firm's home country and each destination. The estimated impact of linguistic distance is small and not statistically significant, while the differences in fixed and sunk costs between a destination with whom Costa Rica has a deep PTA and one with whom it has no agreement are only \$29,000 and \$22,000, respectively. Adding all terms, the gravity terms in fixed costs in, e.g., Mexico, the US, and China, are close to \$100,000, \$125,000, and \$180,000, respectively. For the US and Mexico, these are between the median and the 75 percentile (and below average) of the distribution of observed export revenues in those countries; for China, they are between the 75 and the 95 percentile (and close to the mean). Similarly, the sunk costs estimates in Mexico, the US, and China are close to \$175,000, \$200,000, and \$400,000, respectively, and, thus, larger than the corresponding fixed cost estimates.[20]

The actual fixed costs a firm faces will however differ from the fixed cost gravity component due to the unobserved terms ν_{ijt} and ω_{ijt} and to the effect of export complementarities. As ν_{ijt} is normal and its estimated standard derivation is close to \$81,000, our estimates reveal a large cross-firm heterogeneity in fixed costs in any given country and period. Firms exporting to country j at period t will have on average low values of ν_{ijt}. Thus, actual

[20]In Appendix F.7, we present a figure analogous to Figure 1 but for a model that assumes away the existence of cross-country complementarities. The mean fixed cost function implied by the estimates of the model without complementarities is smaller than that displayed in panel (a) of Figure 1. This is expected, as the estimated fixed costs in the model without complementarities likely approximate an average of the fixed costs faced by different firms depending on their export bundles; e.g., the model without complementarities yields fixed costs estimates in China that are 30% lower than the model with cross-country complementarities.

Figure 1: Estimates of Fixed and Sunk Export Costs

(a) Fixed Export Costs (b) Sunk Export Costs



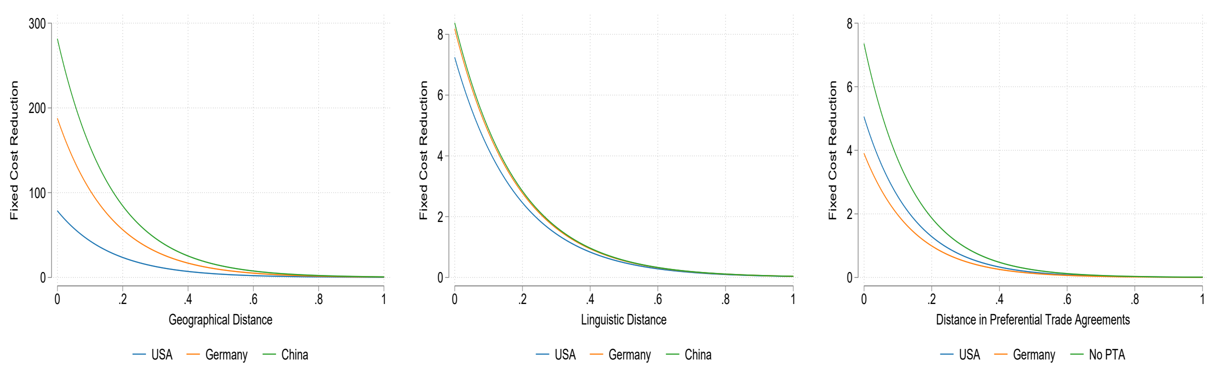
Note: In both figures, countries are identified by their ISO 3166-1 alpha-3 code, and placed in the horizontal axis by their distance to Costa Rica. The vertical axis indicates the estimated cost in thousands of 2010 USD.

exporters will on average face fixed costs that are below what is implied by the fixed cost gravity component, even if they do not export anywhere else in the same period.

In Figure 2, we represent the estimated export complementarities. In each panel, we plot, for the corresponding index x in $\{g, l, a\}$, the function $\hat{\gamma}^E_x(1 + \hat{\varphi}^E_x n^x_{hj}) \exp(-\hat{\kappa}^E_x n^x_{jj'})$ for three countries j (the US, Germany, and China) against their distance to any other country j', $n^x_{jj'}$. Panel (a) shows that complementarities due to geographical proximity are large between countries close to each other (e.g., they reach \$90,000 for countries that are 200 km apart and far from Costa Rica) but decrease quickly, being close to zero between countries whose bilateral distance is 800 km or more. The size of the geographical complementarities is

Figure 2: Estimates of Cross-Country Complementarities By Source

(a) Geographical Proximity (b) Linguistic Proximity (c) Regulatory Proximity



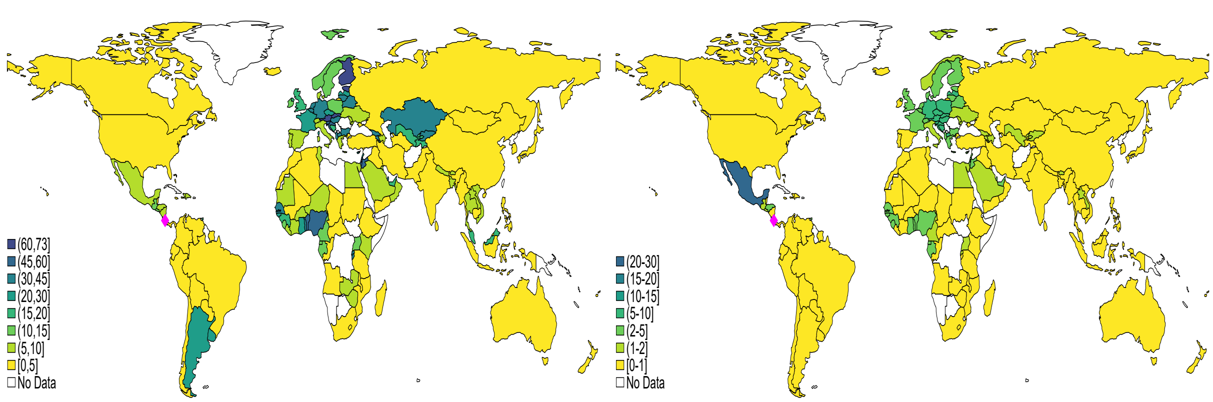
Note: In panels (a) to (c), the horizontal axis corresponds to the distance measures defined in equations (B.1), (B.2), and (1), respectively. The vertical axis indicates the estimated reduction in fixed export costs in thousands of 2010 USD.

heterogeneous across destinations depending on their distance to Costa Rica: for any given distance between two destinations, complementarities are larger in China than in Germany, and in Germany than in the US. Panel (b) shows that linguistic complementarities are always small, reaching a maximum of close to \$8,000 for country pairs whose linguistic distance is zero; i.e., whose residents understand each other with probability one. Finally, panel (c) shows that complementarities due to common participation in PTAs are close to zero unless these agreements are sufficiently deep. Among common members of deep PTAs, the fixed cost reduction in one of them for a firm that exports to the other one varies between \$4,000 and \$7,500 depending on whether the destination shares a PTA with Costa Rica.

In Figure 3, we quantify the cost reductions implied by the estimates in Figure 2. In panel (a), we show for each destination the cost reduction (relative to the gravity component of fixed costs) a firm experiences if it also exports to the country with whom its complementarities are the largest. This reduction is below 5% for countries such as the US or China, but is on average much larger for EU members, being above 45% for several of them. These estimates are due to EU members both sharing a deep PTA and being geographically close to each other. In panel (b), we show there are countries (e.g., Mexico) that, although they do not share strong complementarities with any one country (as shown in panel (a), exporting to Mexico's closest neighbor reduces fixed costs in it in less than 10%), benefit from sharing a moderate level of complementarities with many countries (Mexico shares common language and membership in deep PTAs with many countries). Thus, a firm exporting simultaneously to several countries that share common language or deep PTAs with, e.g., Mexico, may ultimately face small fixed costs in it. Linguistic and regulatory proximity may thus impact firm

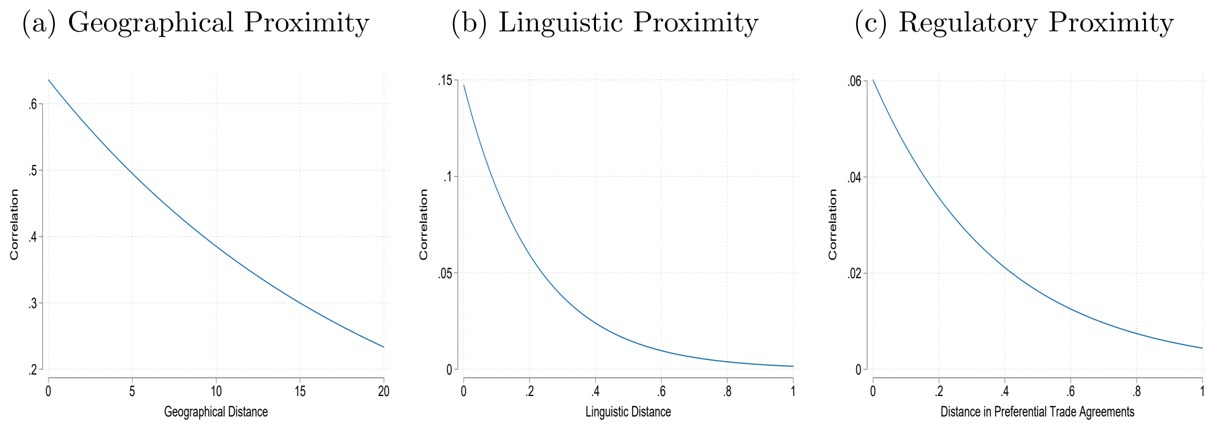
Figure 3: Implications of Estimated Cross-Country Complementarities in Fixed Costs

(a) Cost Reduction from Closest Neighbor (b) Num. Neighbors Reducing Costs in $\geqslant 5\%$



Note: In panel (a), we illustrate for each country j the value $\max_{j'}\{c_{jj't}/g_{jt}\}$. In panel (b), we illustrate for each j the number of other foreign countries $j' \neq j$ for whom $c_{jj't}/g_{jt} \geqslant 5\%$.

Figure 4: Estimates of Correlation Coefficient in Fixed Export Cost Shock By Source



Note: In panels (a) to (c), the horizontal axis indicates the distance measures defined in equations (B.1), (B.2), and (1), respectively. The vertical axis indicates the estimated correlation in ν_{ijt}.

exports even if, as shown in Figure 2, linguistic and regulatory complementarities between any two countries are never large. In Figure F.8, we illustrate the complementarities of the US, China, Germany, and Spain, with all other countries.

In Figure 4, we represent the estimated cross-country correlation in the fixed cost term ν_{ijt} within a firm-period. In each panel, we plot, for the corresponding index x in $\{g, l, a\}$, the function $\gamma_x^N \exp(\kappa_x^N n_{jj'}^x)$ against the distance $n_{jj'}^x$. The key determinant of the correlation coefficient between any two countries is their geographic proximity, although their linguistic proximity also plays a role. Furthermore, these correlation coefficients may be large; e.g., above 0.5 for countries 5000km apart from each other. Thus, when estimating cross-country complementarities, it is important to allow for correlated unobserved export profit shifters. For the US, China, Germany and Spain, we illustrate in Figure F.9 the correlation coefficient of ν_{ijt} vis-a-vis any other country.

Our estimates downplay the importance of export complementarities relative to, e.g., the estimates in Morales et al. (2019), where the reduction in sunk costs in a destination is estimated to be between 70 and 90% for a firm that exported in the previous year to a country that shares border, continent, language, and similar GDP per capita, with that destination. There are several differences between our framework and that in Morales et al. (2019). While our model accounts for firm- and country-specific unobserved determinants of export profits that are potentially correlated across destinations, that in Morales et al. (2019) assumes these determinants away, and this may bias its estimates of export complementarities upwards. Alternatively, complementarities may manifest themselves more prominently in the firm's sunk costs (as considered in Morales et al., 2019) than in fixed export costs, and this may bias our estimates downwards. The reason why we do not allow for cross-country complementarities in sunk costs is that the decisions to export to countries j and j' at period t become substitutes when exporting to country j at t lowers the firm's sunk costs in country j' at $t+1$. However, as far as we know, the literature does not provide an algorithm to solve large-dimensional dynamic models with cross-choice substitutabilities.[21]

Although we limit the heterogeneity in fixed and sunk costs to parametric functions of the geographical, linguistic, and regulatory, distances between countries, we include in Appendix F.9 goodness-of-fit measures that show that our estimated model captures well the observed level and heterogeneity in export probabilities across countries. Specifically, a regression of observed export probabilities by destination on the model-predicted probabilities yields OLS estimates of the constant and slope coefficients equal to 0 and 0.95, respectively, with an associated R^2 equal to 0.86.

7 Quantitative Analysis

In Section 7.1, we quantify the importance of export complementarities by comparing the predictions of versions of the estimated model in which some or all of the cross-country complementarities are set to zero. In Section 7.2, we use the estimated model to compute the impact on Costa Rican arm's length exports of a Brexit-induced increase in the regulatory distance between the UK and current EU members. In Section 7.3, for different counterfactual changes in Costa Rican export barriers, we compare the predictions of our model to those of a re-estimated model that assumes away the presence of complementarities.

7.1 Quantitative Importance of Cross-country Complementarities

To quantify the impact of complementarities, we compute model-implied export choices for each firm and year in the sample using 200 simulations of the vector χ_i of unobserved payoff-relevant variables (see Section 5.2.2). We do so for a baseline model that sets to zero all of the parameters that, according to equation (9), determine the strength of the complementarities, and compare the predictions of such model to that of alternative models that set some or all of these parameters to their estimated values. We report in Table 2 the cross-model differences in the predicted number of firm-country-years with positive exports (export events) and total export revenues. The results in column "All" show that including all complementarities causes the number of export events and total exports to increase in 11.8% and 4.9%, respectively. As shown in the remaining columns, the most important

[21] A specification that allows for dynamic cross-country complementarities while maintaining the supermodularity of the firm's value function is one in which a firm's fixed costs in country j and period t depend on its export choice in country j' at $t-1$. This model may be solved using the algorithm in this paper.

Table 2: Impact of Cross-country Complementarities

	Sources of Complementarities Included:			
Percentage Increase in:	All	Geographic Proximity	Linguistic Proximity	Common Deep PTA
Number of Export Events:	11.76%	7.39%	2.36%	2.55%
Export Revenues:	4.86%	3.03%	0.81%	1.51%

Note: In column *All*, we report the percentage difference in the number of export events and export revenues between a model in which the parameters $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$ are all set to zero and our estimated model. In the other columns, we compare models in which only the subset of these parameters indicated by the corresponding column label is set to their estimated values, while the other ones are kept at zero.

source of complementarities is spatial proximity: setting $(\gamma_g^E, \psi_g^E, \kappa_g^E)$ at their estimated values, while keeping complementarities due to linguistic and regulatory proximity equal to zero, causes export events and total exports to increase in 7.4% and 3%, respectively. Complementarities due to linguistic and regulatory proximity each cause export events to increase in close to 2.5%, and total exports to increase in 0.8% and 1.5%, respectively.

The smaller impact of complementarities on total exports relative to its impact on the number of export events is partly due to complementarities having, all else equal, a larger impact on less attractive destinations. To gain intuition on this model property, consider a setting with two destinations A and B such that, in the absence of complementarities, every firm's potential export profits in A are larger than in B. As shown in Appendix G, introducing complementarities in this context increases exports to B more than to A. The reason is that, without complementarities, exports to A are larger than to B and, with complementarities, firms benefit from a fixed cost reduction in B only if they also export to A. Thus, complementarities push firms to export to both countries, but this implies exports grow more in the country that had a lower level of exports in the absence of complementarities. As large markets are, all else equal, more attractive destinations, complementarities tend to have a larger impact on smaller markets.

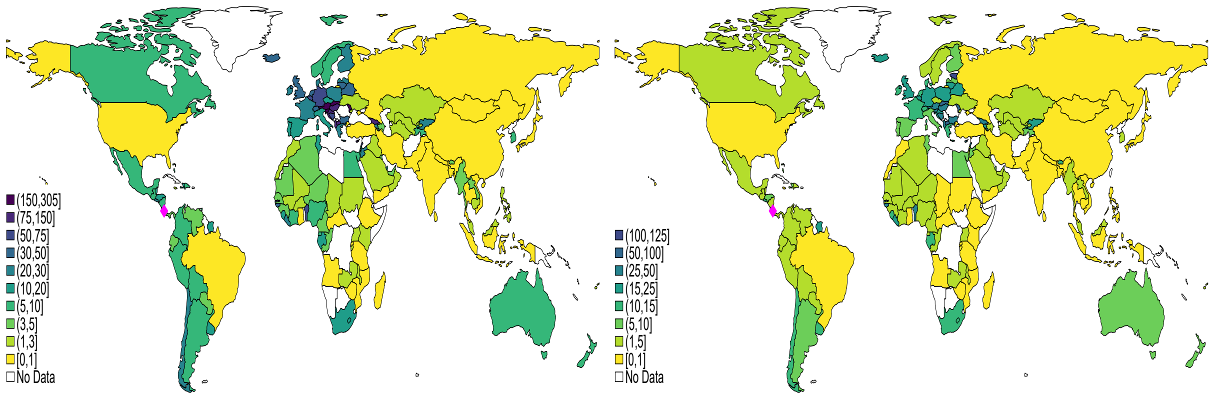
Besides size, the geographical, linguistic, and regulatory proximity of each country to every other country also matters for the impact that complementarities have on exports to it. As a result, as shown in Figure 5, there is a large heterogeneity across countries in the impact of complementarities. In many of them, these play a minimal role; conversely, for some, several of which are located in Central Europe, complementarities increase the number of export events and total exports from Costa Rica in more than 50%. These countries most affected by complementarities are typically small, geographically close to many other destinations, and members of deep PTAs that include many other countries.

Solving our estimated model with complementarities for all firms in the sample takes 125

Figure 5: Impact of Eliminating Cross-country Complementarities

(a) On Number of Export Events

(b) Export Revenues



Note: In Panel (a), we illustrate, for each destination and all firms and years in the sample, the percentage reduction in the total number of firm-year pairs with positive exports predicted by our model when we set the parameters $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$ to zero. In Panel (b), we provide analogous information for the total predicted export revenues.

times more per simulation draw than the model without complementarities, which we solve in 50 seconds. In models with cross-country complementarities, a trade-off between accuracy of model predictions and computational time thus arises. To study this trade-off, we compute the predictions of versions of our model in which we group countries into clusters such that complementarities between countries in the same cluster are fixed to the values implied by our estimates, and complementarities between countries in different clusters are set to zero. Table 3 presents results for different partitions of countries into clusters.[22] Column (1) in Table 3 reproduces column (1) in Table 2. Columns (2) and (3) show that the model predictions are very similar when we partition countries into 2 or 3 clusters; however, the gains in computing time are also small. More interestingly, the time required for computing the model solution decreases in 30% when equating country clusters to continents, and the implied increases in the number of firm-country-periods with positive exports and total exports, relative to the model with no complementarities, are relatively similar to those predicted by our estimated model. Intuitively, as shown in Figure 2, the key source of cross-country complementarities is geographical proximity, and spatial complementarities decay quickly in space, so the assumption that countries located in different continents exhibit no complementarities does not bias model predictions significantly. When partitioning countries into 50 clusters, the computing time is reduced in 75%, and the resulting model still predicts

[22]Given a fixed number of clusters, we follow von Luxburg (2007) to assign countries to clusters. This procedure relies on an adjacency matrix between countries, which we compute using the formula in equation (9) and our estimates of $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$, and uses spectral clustering to categorize countries into groups. In Appendix G.2, we present maps showing the partitions of countries we consider. As illustrated in those maps, our clusters may differ in the number of countries they incorporate.

Table 3: Impact of Cross-Country Complementarities for Alternative Country Clusterings

Percentage Increase in:	(1) Baseline	(2) 2 Clusters	(3) 3 Clusters	(4) 50 Clusters	(5) Continents
Number of Export Events	11.76%	11.58%	11.55%	9.25%	10.24%
Export Revenues	4.86%	4.75%	4.72%	3.30%	4.00%

Note: In each column, we report the percentage difference in the number of export events and export revenues between a model in which the parameters $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$ are all set to zero and alternative country clusters in which we allow for our estimated complementarities within the cluster, and set the complementarities to zero outside of the cluster. The first column corresponds to the baseline case in which we allow for complementarities between all countries. The clusters in columns (2)-(5) are computed by defining an adjacency matrix using our estimates for $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$, and grouping countries using spectral clustering. The clusters in the column "Continents" allow for geographic interdependencies within the continent only. Maps for these clusters are shown in Appendix G.2.

an increase in the number of firm-country-periods with positive exports and total exports, relative to the model with no complementarities, of approximately 9.3% and 3.3%.

7.2 Third-Market Effects of Regulatory Differences Due to Brexit

A potential implication of Brexit is that UK and EU regulations will drift apart. To quantify the third-country effect of this regulatory divergence, we use our estimated model to evaluate the impact on Costa Rican arm's length exports of a permanent increase in 2021 (expected since the 2017 referendum, but unexpected before) in the regulatory distance, $n^a_{jj't}$, between the UK and all EU members from zero (its pre-Brexit value) to one (its maximum value). Specifically, for all sample firms and these two regulatory distances, we compute model-implied export choices for 200 simulations of the vector χ_i, and report in Table 4 the relative differences in the expected number of export events and total exports.

Model-predicted exports to the UK fall due to the increased regulatory distance between the UK and the EU. Specifically, the predicted fall in export events and total exports in the 10 years after Brexit is around 5%. In the four years between the Brexit referendum and the UK's effective EU withdrawal, firms anticipate the policy change, and the number of export events and total exports to the UK fall in 1.6% and 0.8%, respectively.

Although the reduction in complementarities between the UK and the EU is symmetric, the effect on exports to the UK is larger than that on exports to the EU, where the drop is around 0.4%. Zooming in on individual EU members, our model predicts that the countries geographically close to the UK will be more affected than those further away; e.g., in comparison to the 0.4% reduction in overall exports to the EU between 2021 and 2030, exports fall in 1.3% and 1% in Belgium and Ireland, respectively. To understand these effects, one should bear in mind that the cross-country complementarities in the estimated model imply that the reduction in exports to the UK as a result of its regulatory isolation from the EU

Table 4: Impact of Regulatory Differences Due to Brexit

Countries:	Percentage Reduction in: Export Events		Export Revenues	
	2017-20	*2021-30*	*2017-20*	*2021-30*
United Kingdom	-1.63%	-4.57%	-0.82%	-5.34%
European Union	-0.17%	-0.44%	-0.06%	-0.39%
In particular:				
Belgium	-0.48%	-1.62%	-0.10%	-1.33%
Ireland	-0.22%	-0.95%	-0.10%	-1.01%

Note: For the geographic area indicated in the column "Countries," we report the relative change for the periods 2017-20 and 2021-30 in the number of export events and total exports of all sample firms caused by a permanent change in 2021 (expected since 2017) in the regulatory distances between the UK and every EU member from zero to one.

will have subsequent effects on countries geographically close to the UK, such as Belgium and Ireland. Similarly, exports to countries with large English-speaking populations will also be affected by the increase in the UK-EU regulatory distance, but these effects are small as a result of linguistic complementarities being small (see Section 6.2).

Without complementarities, a partial equilibrium model (such as ours) predicts Costa Rican exports are unaffected by changes in trade barriers (regulatory or otherwise) between destinations. General equilibrium models à la Eaton and Kortum (2002) or Anderson and van Wincoop (2003) imply exports of different origins are substitutes and, thus, predict Costa Rican exports to the UK and the EU to *increase* in reaction to the raise in the UK-EU trade barriers.[23] The third-market effects implied by cross-country complementarities in our model are thus of opposite sign to those in standard trade models.

7.3 Impact of Reductions in Export Tariffs

In 2022, Costa Rica applied for CPTPP membership. We evaluate the effect on Costa Rican exports of a reduction in export tariffs to this trade bloc. Specifically, for the period 2022-37, all sample firms, and 200 simulations of the vector χ_i, we compute model-implied exports in a setting in which tariffs do no change and in one in which, from 2022 onwards, Costa Rican export tariffs to CPTPP members are zero. We do so using our estimated model and a re-estimated model analogous to ours but in which complementarities are assumed away (see Appendix F.7). To provide some guidance on when the model without complementarities

[23] Adão et al. (2017) and Lind and Ramondo (2023) allow for more flexible elasticities of substitution across export countries, but maintain the assumption that different export countries are substitutes. For a framework that allows for positive third-market effects, see Fajgelbaum et al. (2023).

Table 5: Impact on Trade Area Members of Eliminating Export Tariffs to Them

Model With Cross-Country Complementarities				Model Without Cross-Country Complementarities			
Tariff Changes With CPTPP		Tariff Changes With The EU		Tariff Changes With CPTPP		Tariff Changes With The EU	
Export Events	Export Revenues	Export Events	Export Revenues	Export Events	Export Revenues	Export Events	Export Revenues
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
15.69%	29.93%	65.33%	83.08%	14.57%	28.89%	54.90%	79.58%

Note: The results in columns (1) to (4) are computed using our estimated model; those in columns (5) to (8) are computed using the model described in Appendix F.7. The results in columns (1), (2), (5), and (6) report the impact of eliminating Costa Rican export tariffs to all CPTPP members; those in columns (3), (4), (7), and (8) evaluate the impact of eliminating tariffs with all EU members. Results aggregate predictions for all sample firms, the period 2022-37, and 200 draws of χ_i.

generates quantitatively different predictions from those of our estimated model, we also evaluate the effect on Costa Rican exports of eliminating export tariffs to the EU.

As shown in columns (1) and (2) in Table 5, the estimated model predicts the number of firm-year pairs with positive exports and total exports to CPTPP members to increase in 16% and 30%, respectively, when tariffs are brought to zero. Columns (5) and (6) show that researchers using a model analogous to ours but in which cross-country complementarities are assumed away would predict a growth in Costa Rican exports to CPTPP members only slightly smaller than that predicted by our model. Furthermore, the model with complementarities predicts minimal export growth to non-CPTPP countries, matching thus closely the zero export growth to these destinations predicted by the model without complementarities. The reason why the predictions of the models with and without complementarities are so similar is that current CPTPP members exhibit small estimated complementarities both with each other and with non-members. Thus, the growth in exports to any member country has small spillovers on other countries.

When computing the impact of Costa Rica signing a PTA with the EU that sets its export tariffs to zero in all member countries, the estimated model predicts the number of export events and total exports to member countries to grow in 65% and 83%, respectively. The re-estimated model without complementarities predicts these growth rates to be 55% and 80%. Importantly, the model with complementarities differs from the model without complementarities in that the former predicts significant export growth to countries that are not EU members (thus, whose tariffs do not change in the counterfactual exercise) but that are geographically close to some EU members, or that share a deep PTA with them; e.g., the predicted export growth among the Balkan countries that do not belong to the EU is above 10%, the export growth in Great Britain, Switzerland, and Iceland, is close to 7%, and that in Lebanon and Tunisia is around 3%. The reason for the disparity in model predictions in this case is that EU members exhibit strong complementarities between themselves and with other countries and, thus, export growth to an EU member may have important spillovers on other destinations. The model without complementarities assumes away these spillovers and, thus, predicts smaller changes in exports to many destinations.

8 Conclusion

We solve a partial-equilibrium firm export dynamics model featuring cross-country complementarities. In our model, the firm has rational expectations and chooses every period the bundle of export destinations that maximizes its expected discounted sum of profits. We introduce an algorithm to solve the firm's combinatorial dynamic discrete choice problem. Our algorithm may be used more generally to solve single-agent dynamic entry problems that exhibit complementarities in the entry decisions both across markets and over time.

Our estimates reveal substantial heterogeneity in complementarities across country pairs. While certain countries (e.g., the US or China) appear isolated according to our estimates, several groups of countries (the EU in particular) are closer to a single market than to a collection of unconnected markets, questioning the standard definition of countries as independent export markets.

We predict Costa Rica's arm's length exports are close to 5% larger due to the estimated cross-country complementarities. We quantify the impact Brexit has on Costa Rican exports to the UK and the EU as a result of both countries no longer sharing a deep PTA: although trade barriers between Costa Rica and every foreign country are held constant in this exercise, exports to the UK and the EU drop in 5% and 0.4%, respectively, illustrating that deep PTAs may give rise to positive trade *creation* effects. Finally, we show that researchers that assume away the presence of complementarities when predicting the impact of counterfactual changes in trade policy will obtain predictions similar to those of our estimated model when the policy changes affect isolated countries, and potentially quite different predictions when the policy changes affect countries that exhibit important complementarities with other destinations.

Our paper is a step towards merging two literatures, the literature on firm export dynamics, which has a long tradition within international trade, and a more recent literature exploring interdependencies across choices in firm decisions. Natural next steps are to allow for additional sources of cross-choice interdependencies (e.g., increasing marginal production costs), to study the impact of complementarities in dynamic general-equilibrium frameworks, or to quantify the mechanisms that give rise to these complementarities.

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A Optimization Problem and Solution Algorithm

In Appendix A.1, we describe an optimization problem that covers that in equation (18). In Appendix A.2, we propose a solution algorithm that covers that in Section 4 and Appendix D.1, and state several of its properties if applied to the problems described in Appendix A.1.

A.1 Optimization Problem

To simplify the notation and without loss of generality, consider an agent born at period $= 0$. In every period $t \geqslant 0$, this agent makes J simultaneous binary choices with the goal of maximizing the expected discounted sum at birth of infinite per-period (static) payoffs.

Per-period payoffs in any t depend on a shock ω_t that takes values in a set Ω_t according to a distribution $Q_t(\omega_t|\omega_{t-1}, \ldots, \omega_0)$. We denote as $z^t = \{\omega_{t'}\}_{t'=0}^{t}$ the history of shocks in all periods $t' \leqslant t$, and as $Z^t = \times_{t'=0}^{t}\Omega_{t'}$ the set of all possible period-t histories. We denote as $y_j(z^t) \in \{0,1\}$ a generic choice at z^t for alternative j, as $y(z^t) \in \{0,1\}^J$ a generic vector of choices at z^t for all J alternatives, and as $\boldsymbol{y} \in Y$ a generic vector of choices for all $t \geqslant 0$, all $z^t \in Z^t$, and all alternatives:

$$Y = \times_{t=0, z^t \in Z^t}^{\infty}\{0,1\}^J. \tag{A.1}$$

Considering only optimization problems where the solution exists and is unique, we can write

$$\boldsymbol{o} = \underset{\boldsymbol{y} \in Y}{\operatorname{argmax}}\, \Pi_0(\boldsymbol{y}), \tag{A.2}$$

where $\Pi_0(\boldsymbol{y})$ is the agent's objective function and $\boldsymbol{o}$ is the optimal choice for all $t \geqslant 0$ and all $z^t \in Z^t$. Thus, using $o(z^t)$ to denote the agent's optimal choice at z^t, it holds

$$\boldsymbol{o} = \{o(z^t)\}_{t=0, z^t \in Z^t}^{\infty}. \tag{A.3}$$

The following assumption establishes a list of conditions on $\Pi_0(\cdot)$.

Assumption 1 *Assume:*

1. *(Additive separability of static profits) The function* $\Pi_0(\cdot)$ *satisfies*

$$\Pi_0(\boldsymbol{y}) = \pi_0(y(z^0), 0_J, \omega(z^0)) + \sum_{t=1}^{\infty} \delta^t \mathbb{E}\big[\pi_t(y(z^t), y(z^{t-1}), \omega(z^t))\big], \tag{A.4}$$

where the expectation is over $\{z^t\}_{t=1}^{\infty}$, $\delta \in (0,1)$ *and, for all* $t \geqslant 0$,

$$\pi_t(y(z^t), y(z^{t-1}), \omega(z^t)) = \sum_{j=1}^{J} \big(\hat{\pi}_{jt}(y_j(z^t), y_j(z^{t-1}), \omega(z^t)) + \tilde{\pi}_{jt}(y(z^t), y(z^{t-1}))\big) \tag{A.5}$$

where $\hat{\pi}_{jt}\colon \{0,1\} \times \{0,1\} \times \Omega_t \longrightarrow \mathbb{R} \cup \{-\infty\}$ *and* $\tilde{\pi}_{jt}\colon \{0,1\}^J \times \{0,1\}^J \longrightarrow \mathbb{R}$.

2. *(Supermodularity) For all* $t \geqslant 0$ *and* $\omega_t \in \Omega_t$, π_t *is supermodular in* $(y(z^t), y(z^{t-1}))$ *on* $\{0,1\}^J \times \{0,1\}^J$.

3. *(Inaction) For all $j \in [1, J]$, $t \geqslant 0$, and $z^t \in Z^t$, $\exists\ y_j(z^t) \in \{0, 1\}$ such that, defining the set $X_t \equiv \{0, 1\} \times \Omega_t$, it holds that $\hat{\pi}_{jt}(y_j(z^t), x) \geqslant -K$ for all $x \in X_t$ and a $K \in \mathbb{R}_{\geqslant 0}$.*
4. *(Markov with finite state space) For all $t \geqslant 0$, Ω_t is finite and $Q_t(\omega_t|\omega_{t-1}, \ldots, \omega_0) = Q_t(\omega_t|\omega_{t-1})$.*
5. *(Stationarity) There exists T such that, for all $t \geqslant T$ and all $j \in [1, J]$, $\Omega_t = \Omega_T$, $Q_t(\cdot) = Q_T(\cdot)$, $\hat{\pi}_{jt} = \hat{\pi}_{jT}$ and $\tilde{\pi}_{jt} = \tilde{\pi}_{jT}$.*

As shown in Appendix E.1, equating agents to firms and alternatives to potential export destinations, the model described in Section 3 satisfies all restrictions in Assumption 1.

A.2 Solution Algorithm

We describe here an iterative algorithm that yields upper bounds on the solution to the problem in equation (A.2) if the function $\Pi_0(\cdot)$ satisfies the restrictions listed in Assumption 1. An algorithm that yields lower bounds may be similarly formulated.

As a preliminary step, partition the J alternatives into U groups indexed by u. Denote as $M_u \subseteq \{1, \ldots, J\}$ the set of alternatives included in group u, and denote as M_u^c the complement of M_u. E.g., if $J = 4$ and $U = 3$, we can form the subsets $M_1 = \{1, 2\}$, $M_2 = \{3\}$, and $M_3 = \{4\}$, and the corresponding complements are $M_1^c = \{3, 4\}$, $M_2^c = \{1, 2, 4\}$, and $M_3^c = \{1, 2, 3\}$. For each set M_u and each iteration $n = 1, 2, 3, \ldots$ of the algorithm, we solve

$$\bar{\boldsymbol{o}}_{M_u}^{(n)} = \underset{y_{M_u} \in Y_{M_u}}{\operatorname{argmax}} \, \Pi_0(\boldsymbol{y}_{M_u}, \bar{\boldsymbol{y}}_{M_u^c}^{(n)}), \tag{A.6}$$

where $\boldsymbol{y}_{M_u}$ is a generic vector of choices in every alternative in M_u, all periods $t \geqslant 0$, and every history z^t that may be reached at t. The set Y_{M_u} includes all feasible values of $\boldsymbol{y}_{M_u}$:

$$Y_{M_u} = \times_{t=0, z^t \in Z^t}^{\infty} \{0, 1\}^{J_u},$$

where J_u is M_u's cardinality. The second argument of $\Pi_0(\cdot)$ in (A.6) is an upper bound on the firm's optimal choice in all alternatives not in M_u, all $t \geqslant 0$, and all $z^t \in Z^t$:

$$\bar{\boldsymbol{y}}_{M_u^c}^{(n)} = \{\bar{y}_{M_u^c}^{(n)}(z^t)\}_{t=0, z^t \in Z^t}^{\infty}, \qquad \text{with} \qquad \bar{y}_{M_u^c}^{(n)}(z^t) \geqslant o_{M_u^c}(z^t) \text{ for all } t \geqslant 0 \text{ and } z^t \in Z^t,$$

with $o_{M_u^c}(z^t)$ the vector of optimal choices at t and history z^t in all alternatives not in M_u.

Solving the problem in equation (A.6) for any group u at any iteration n requires specifying first the upper-bounds included in the vector

$$\bar{\boldsymbol{y}}_{M_u^c}^{(n)}.$$

For computational reasons, we set the upper bound corresponding to any country j, period t, and history z^t, to a value that does not vary across histories; i.e., we set

$$\bar{y}_j^{(n)}(z^t) = \bar{b}_{jt}^{(n)} \text{ for all } z^t \in Z^t. \tag{A.7}$$

In the first iteration (i.e., for $n = 1$), we set each of these upper bounds to its largest value within the feasible choice set; i.e., for every j and $t \geqslant 0$, we set

$$\bar{b}_{jt}^{(1)} = 1. \tag{A.8}$$

In all subsequent iterations (for all $n > 1$), we set

$$\bar{b}_{jt}^{(n)} = \max_{z^t \in Z^t} \bar{o}_j^{(n-1)}(z^t), \tag{A.9}$$

where $\bar{o}_j^{(n-1)}(z^t)$ is the element for alternative j, period t, and history z^t of the vector $\bar{\boldsymbol{o}}_{M_u}^{(n-1)}$ for the set M_u including j. Equation (A.9) shows that, to compute the iteration-n upper bound on the firm's optimal choice in j at history z^t, we use the outcome of the optimization problem in equation (A.6) at iteration $n-1$ for the set M_u including j. Specifically, as shown in equation (A.9), we assign to every j, t, and z^t, the max of the outcomes obtained for j and t across all $z^t \in Z^t$.

Theorem 1 establishes properties of the algorithm defined in equations (A.6) to (A.9)

Theorem 1 *Let $\bar{b}_{jt}^{(n)}$ be defined by equations* (A.6) *to* (A.9)*, and let $o_j(z^t)$ be the element of the vector $\boldsymbol{o}$ defined in equation* (A.2) *that corresponds to alternative j and history z^t. Then, for all $j = 1, \ldots, J$, $t = 1, 2, \ldots$, $z^t \in Z^t$, and $n = 1, 2, 3, \ldots$, it holds that*

1. *$\bar{b}_{jt}^{(n)} \geqslant o_j(z^t)$.*
2. *$\bar{b}_{jt}^{(n)} \leqslant \bar{b}_{jt}^{(n-1)}$.*
3. *There exists $N < \infty$ such that $\bar{b}_{jt}^{(n)} = \bar{b}_{jt}^{(n-1)}$ for all $n \geqslant N$.*

Theorem 1 states that the values $\{\bar{b}_{jt}^{(n)}\}_{j=1,t=0}^{J,\infty}$ computed according to equations (A.6) to (A.9) are an upper bound on the firm's optimal choice at every history, get tighter with every iteration, and converge after a finite number of iterations. See Appendix E for a proof of Theorem 1.

Property 3 of Theorem 1 does not imply that the upper bound defined by equations (A.6) to (A.9) converges to the solution of the firm's problem in equation (A.2). However, as the partition of the J alternatives into U subgroups gets coarser, the upper bound defined by equations (A.6) to (A.9) gets tighter. In the limiting case in which $U = 1$ and, therefore, $M_u = \{1, 2, \ldots, J\}$, the problem in equation (A.6) coincides with that in equation (A.1).

The algorithms implemented in each of the steps described in Section 4 and Appendix D.1 are special cases of the algorithm defined in equations (A.6) to (A.9). E.g., the algorithm implemented in step 1 is a case in which: (a) $U = J$ and, for $u = 1, \ldots, J$, the set M_u is a singleton; and (b) period $t = 0$ corresponds to the birth year of the firm (i.e., $t = \underline{t}_i$). The algorithm implemented in step 5 is a case in which: (a) $U < J$, and for some $u = 1, \ldots, U$, the set M_u includes more than one country; and, (b) period $t = 0$ corresponds to the first period at which the upper and lower bounds computed in the previous step differ.

Online Appendix for "Firm Export Dynamics in Interdependent Markets"

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Contents

B Additional Reduced-Form Results

B.1 Firm-level Data: Sample Descriptive Statistics

We provide here descriptive statistics for the firm-level data introduced in Section 2. In Table B.1, we report information for every sample year on total manufacturing exports, total number of exporting firms, and total number of foreign countries to which manufacturing firms exported in the corresponding year. While the total number of exporters remained stable at a number between approximately 400 and 450, and the total number of export destinations remained stable at around 90 destinations, the total export volume grew significantly in real terms between 2005 and 2015.

Table B.1: Aggregate Statistics

Years	Total Exports	Number of Exporters	Number of Destinations
2005	262,549.6	400	95
2006	303,344.6	415	96
2007	332,929.1	422	91
2008	371,202.9	419	91
2009	328,435.2	438	87
2010	347,235.1	432	96
2011	431,820.7	456	91
2012	479,806.0	459	90
2013	450,472.3	437	84
2014	494,083.5	436	84
2015	479,485.1	395	90

Notes: *Total Exports* are reported in thousands of 2013 US dollars.

In Table B.2, we report the mean and median domestic sales across all firms and across exporters. As in datasets similar to ours, the distribution of domestic sales is skewed to the right

Table B.2: Firm-level Statistics

Years	Domestic Sales (All Firms)		Domestic Sales (Exporters)		Exports		Number of Destinations (Exporters)		
	Average	Median	Average	Median	Average	Median	Average	Median	95th/99th perc.
2005	684.4	119.4	3,312.0	822.9	656.4	63.4	3.38	2	10/17
2006	695.4	118.4	3,553.2	772.6	731.0	63.1	3.28	2	10/18
2007	782.4	131.7	3,864.6	904.3	788.9	63.7	3.35	2	10/16
2008	889.6	147.0	4,693.6	1,160.0	885.9	66.4	3.30	2	9/18
2009	839.1	126.4	4,682.5	1,033.4	749.9	43.4	3.19	2	10/18
2010	937.2	139.2	5,256.7	1,161.1	803.8	56.7	3.28	2	9/18
2011	1,031.9	147.4	5,601.4	1,201.7	947.0	56.3	3.25	2	9/19
2012	1,067.5	154.1	5,663.2	1,091.7	1,045.3	65.9	3.22	2	9/19
2013	1,098.9	158.1	5,922.9	1,178.6	1,030.8	78.2	3.35	2	10/17
2014	1,043.8	147.4	5,793.3	1,208.3	1,133.2	59.7	3.28	2	10/18
2015	1,166.0	155.8	6,809.5	1,566.5	1,213.9	80.5	3.62	2	11/20

Notes: *Domestic sales* and *Exports* are reported in thousands of 2013 US dollars. We measure domestic sales by subtracting total export revenue (from the Customs dataset) from total revenue.

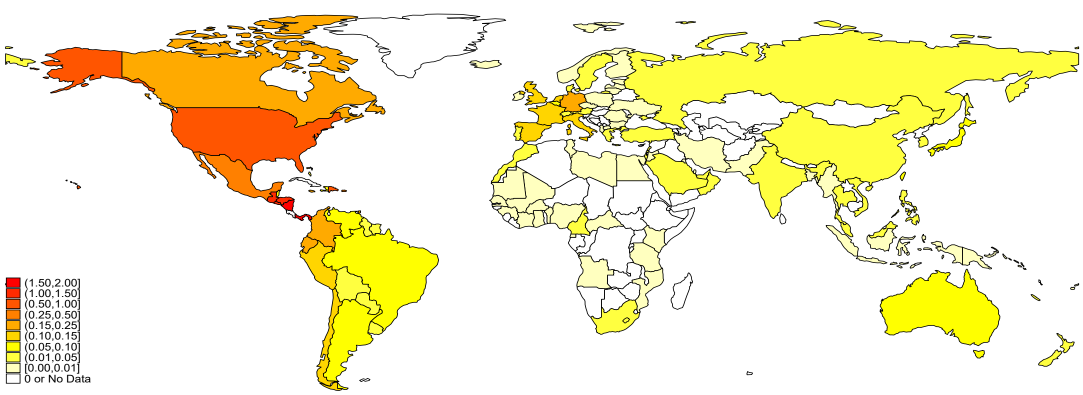
(mean domestic sales are larger than median domestic sales), and exporters are larger on average than non-exporters (mean domestic sales among exporters are larger than in the overall population). We also report in Table B.2 mean and median export revenues. Consistently with the fact that, between 2005 and 2015, total exports grew significantly while the number of exporters remained roughly constant, mean export revenues also grew markedly during the same period.

The last three columns in Table B.2 describe the distribution of the number of export destinations across firms. Three features are salient. First, it is very skewed: the difference in the number of destinations between the median exporter and that at the 95th percentile is similar to the difference between the exporter at the 95th percentile and that at the 99th percentile. Second, some firms export to many destinations; the 95% percentile is close to 10, and the 99th percentile oscillates between 17 and 20. Third, the distribution is stable over time.

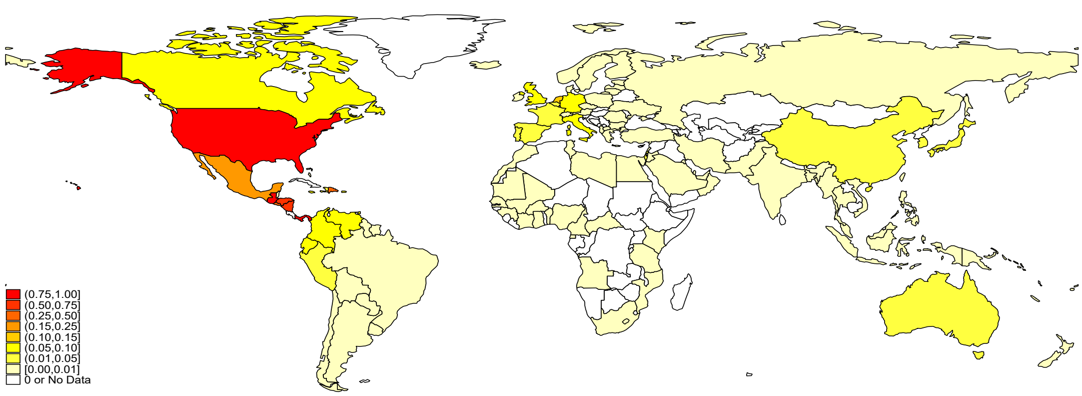
The maps in Figure B.1 show the total number of export events (i.e., firm-year pairs with positive exports) and the total volume of exports by destination, in both cases relative to the corresponding magnitude in the US. Both maps show that the most popular destinations are countries in North and Central America, followed by China, Australia, and countries in Europe. The top 5 destinations by export revenue are the US, Guatemala, Panama, Nicaragua and Honduras.

Figure B.1: Export Activity by Destination Country During Period 2005-2015

(a) Total Number of Export Events



(b) Total Volume of Exports



Notes: Panel (a) shows the total number of firm-year pairs with positive exports relative to that in the US. Panel (b) shows the total volume of manufacturing exports relative to that in the US.

In Table B.3, we present the mean and percentiles of the distribution of annual firm-level exports to several countries during 2005-2015. This distribution is disperse and skewed to the right.

Table B.3: Distribution of Export Sales in Several Markets

Country	Average	Percentile					
		5	25	50	75	95	99
United States	597.6	0.4	5.0	28.1	227.4	3,477.9	9,615.9
Panama	271.4	1.2	7.4	32.5	138.6	1,013.6	5,022.9
Germany	350.8	0.3	6.3	54.0	419.5	1,844.9	3,015.5
Nicaragua	209.8	1.2	8.7	37.6	134.5	879.5	3,013.9
Mexico	295.4	0.4	9.0	51.0	284.2	1,224.8	2,637.1
China	128.8	0.2	3.9	21.8	68.9	713.7	1,584.0

Notes: All numbers in this table are reported in thousands of 2013 dollars.

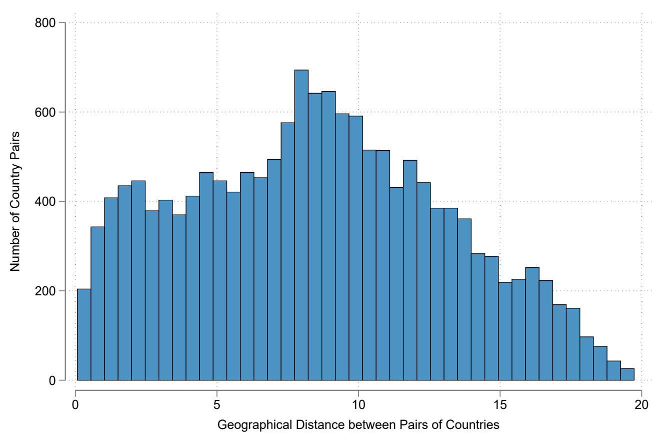
B.2 Geographical Distance

We measure the geographic distance $n^g_{jj'}$ between any two countries j and j' as

$$n^g_{jj'} \equiv \Big(\sum_{k \in j} \sum_{k' \in j'} \frac{pop_k}{pop_j} \frac{pop_{k'}}{pop_{j'}} (dist_{kk'})^{-1} \Big)^{-1}, \tag{B.1}$$

where k and k' respectively index cities in countries j and j', pop_k and $pop_{k'}$ denote k and k' population, pop_j and $pop_{j'}$ denote the total population of the cities in countries j and j' used to calculate $n^g_{jj'}$, and $dist_{kk'}$ is the distance between k and k' in thousands of kilometers. In Figure B.2, we present a histogram of the distance measure in equation (B.1) across country pairs.

Figure B.2: Histogram of Bilateral Geographic Distances

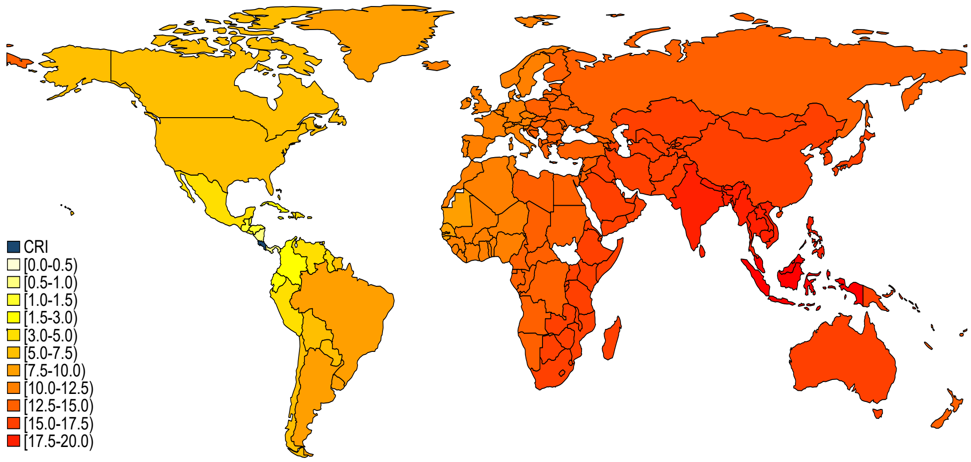


Notes: The vertical axis indicates the number of country pairs whose geographical distance $n^g_{jj'}$ falls in the corresponding bin. The horizontal axis denotes geographical distance in thousands of kilometers.

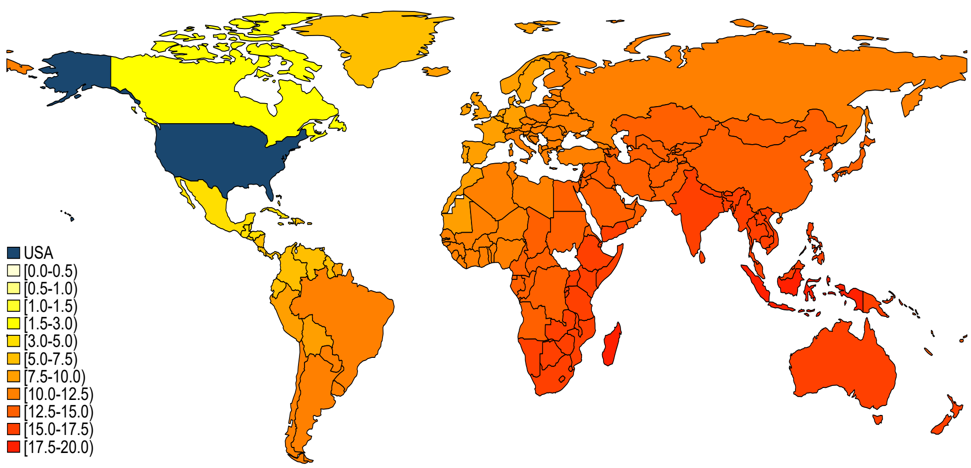
In Figure B.3, we represent in maps the geographical distance from Costa Rica (in Figure B.3a), the United States (in Figure B.3b), France (in Figure B.3c) and China (in Figure B.3d), respectively, to any other country of the world.

Figure B.3: Geographical Distances From Certain Countries

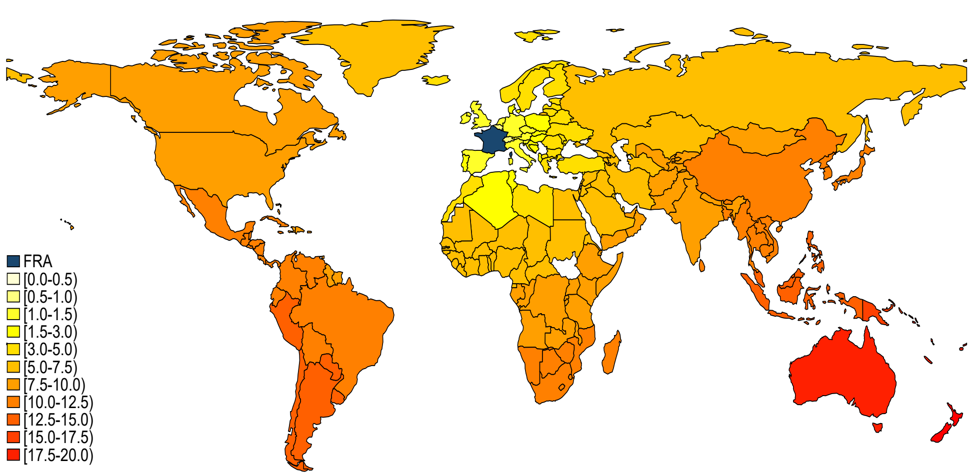
(a) From Costa Rica



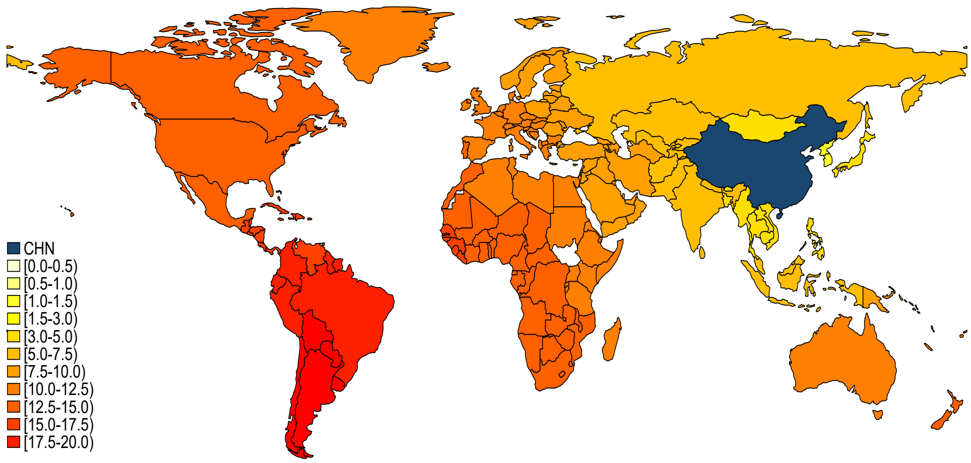
(b) From the United States



(c) From France



(d) From China



Notes: Each panel indicates the geographical distance (computed using the expression in equation (B.1)) between a particular country (Costa Rica in panel (a), the US in panel (b), France in panel (c), and China in panel (d)) and any other country in the world. Distances are reported in thousands of kilometers.

B.3 Linguistic Distance

We measure the linguistic distance $n^l_{jj'}$ between any two countries j and j' as

$$n^l_{jj'} \equiv \max\Big\{0, 1 - \sum_{k=1}^{K} s_{jk}s_{j'k}\Big\}, \tag{B.2}$$

where s_{jk} is the share of country j's population that speak language $k = 1, \ldots, K$. To obtain a list of languages and information on the population shares s_{jk} that speak a language k in a country j, we use the Ethnologue dataset (see Desmet et al., 2012, for an application of Ethnologue data).

Ethnologue defines languages according to 15 aggregation levels; e.g., at the 1st level, all Indo-European languages are considered the same language; at the 15th level, Spanish and Extremaduran are distinct. We use the 9th level, the first one classifying Portuguese and Spanish as distinct.

Ethnologue provides information by country on the population share that speaks a language as first language, and on the population share that speaks it as second language, but it does not provide information on the distribution of second language speakers conditional on their first language. The measure in equation (B.2) assumes a joint distribution of first and second languages such that the linguistic distance between any two countries is minimized. To illustrate this, consider a setting with only two languages, k_1 and k_2. In this setting, the probability that two individuals i and i' randomly selected from countries j and j', respectively, speak a common language is:

$$\begin{gathered} P((\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cup (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})) \\ = \\ P(\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) + P(\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\}) - \\ P((\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cap (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})). \end{gathered}$$

Using the notation in equation (B.2), we can rewrite this expression as

$$P((\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cup (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})) =$$

$$s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} - P((\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cap (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})).$$

Thus, we can rewrite the probability that i and i' do not speak a common language as

$$1 - s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2} + P((\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cap (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})).$$

As the Ethnologue data does not contain information on the joint distribution of first and second languages spoken within a country, we cannot compute

$$P((\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cap (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})).$$

We can however obtain a lower bound $LB_{jj'}$ on this probability as

$$LB_{jj'} = \begin{cases} 0 & \text{if } s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2} \leqslant 1, \\ s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} - 1 & \text{if } s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} > 1, \end{cases}$$

or, equivalently, $LB_{jj'} = \max\{0, s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} - 1\}$. We thus obtain a lower bound on the probability that i and i' do not speak a common language as

$$1 - s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2} + LB_{jj'} = 1 - s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2} + \max\{0, s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} - 1\}$$

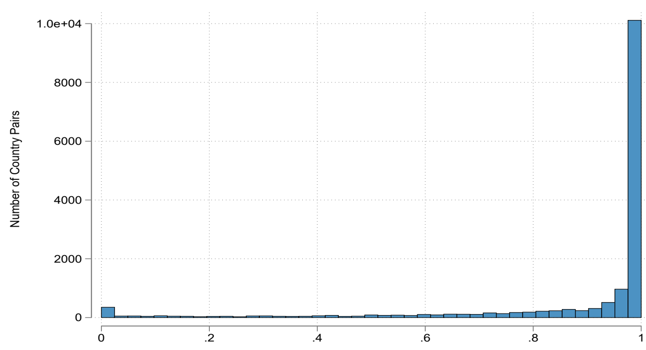
or, equivalently,

$$\max\{0, 1 - s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2}\}.$$

This expression corresponds to that in equation (B.2) for the case with two languages, k_1 and k_2.

In Figure B.4, we present a histogram of bilateral linguistic distances. For most country pairs, a randomly selected resident of one of the two countries will not share any language with a randomly selected resident of the other country. Thus, for most country pairs, their linguistic distance equals one, which is the maximum possible value of the distance measure introduced in equation (B.2).

Figure B.4: Histogram of Bilateral Linguistic Distances

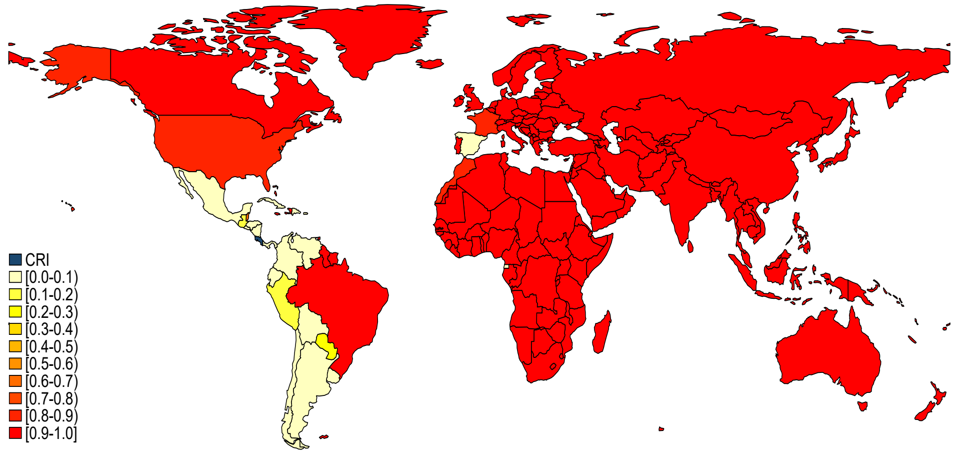


Notes: The vertical axis indicates the number of country pairs whose linguistic distance $n^l_{jj'}$ falls in the corresponding bin. The horizontal axis denotes the corresponding linguistic distance.

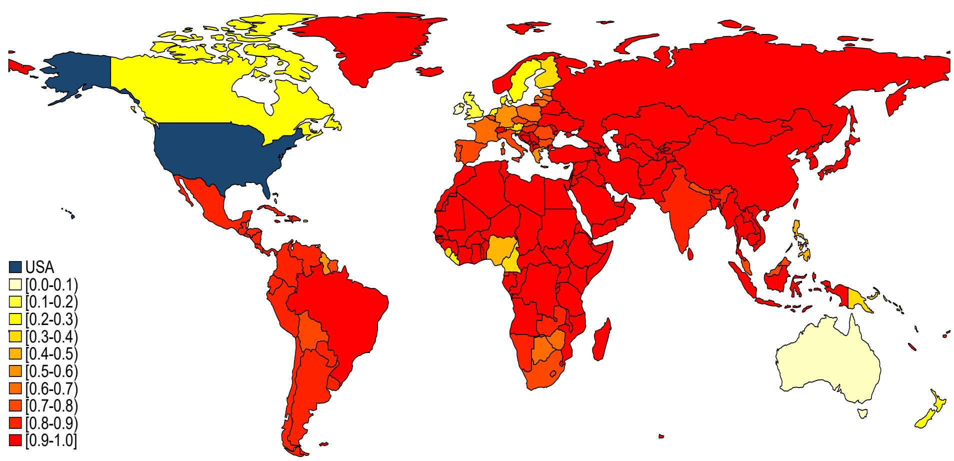
In Figure B.5, we represent bilateral linguistic distance measures from Costa Rica (in Figure B.5a), the US (in Figure B.5b), France (in Figure B.5c) and China (in Figure B.5d) to any other country of the world.

Figure B.5: Bilateral Linguistic Distances From Certain Origin Countries

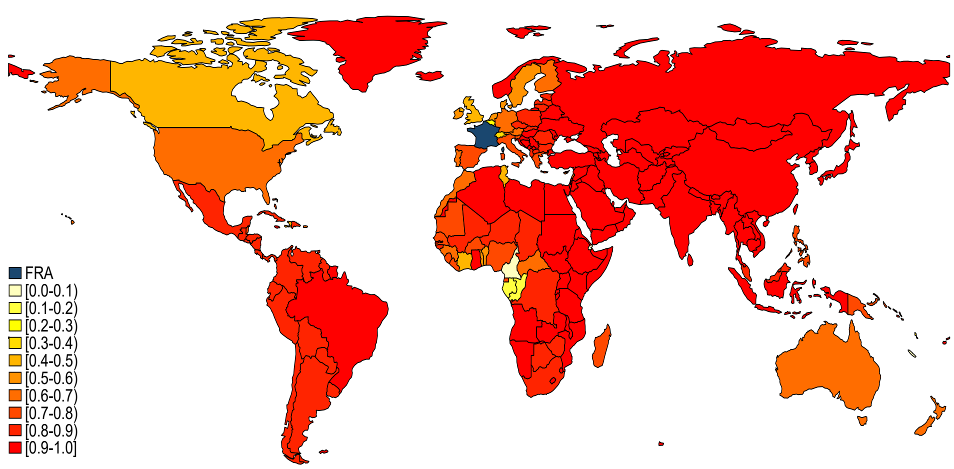
(a) From Costa Rica



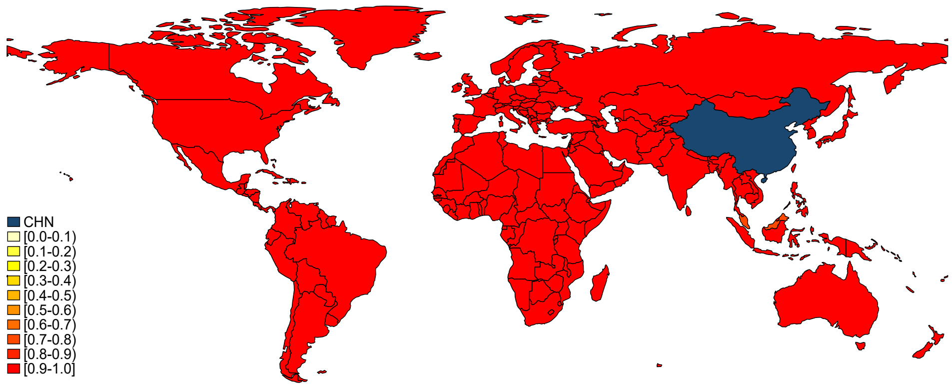
(b) From the United States



(c) From France



(d) From China



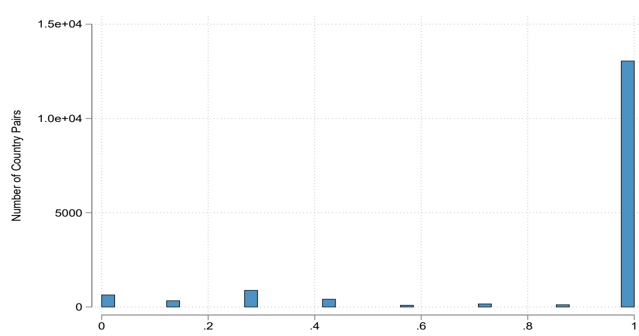
Notes: Each of the four panels in this figure indicate the linguistic distance (computed according to the expression in equation (B.2)) between a particular country (Costa Rica in panel (a), the US in panel (b), France in panel (c), and China in panel (d)) and any other country in the world.

The map in panel (a) reflects the large set of countries where Spanish is commonly spoken. The map in panel (b) shows that, as a consequence of the popularity of English as second language in many European countries, countries such as the Netherlands, Denmark, or Sweden, appear linguistically close to the US. Interestingly, as panel (c) reveals, the popularity of English as second language makes pairs of countries where none of them have English as official language (e.g., France and Sweden, France and Denmark) linguistically close. Finally, panel (d) shows that China, whose residents largely speak neither English nor Spanish, is linguistically isolated.

B.4 Measures of Regulatory Distance

In Figure B.6, we present a histogram of the regulatory distance measure introduced in equation (1). As this figure reveals, most country pairs do not share any PTA containing a provision in at least one of the policy areas listed in footnote 10.

Figure B.6: Histogram of Bilateral Distances in PTAs

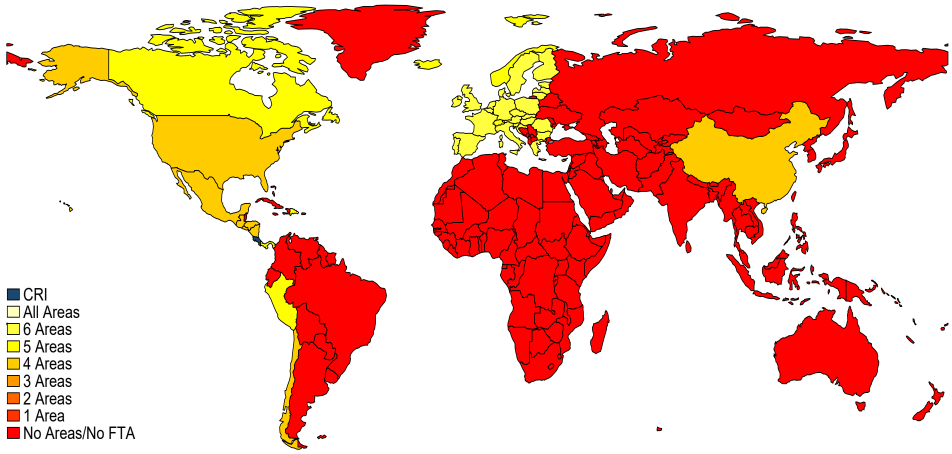


Notes: The vertical axis indicates the number of country pairs whose distance $n^a_{jj'}$ falls in the corresponding bin. The horizontal axis denotes the value of the corresponding distance measure.

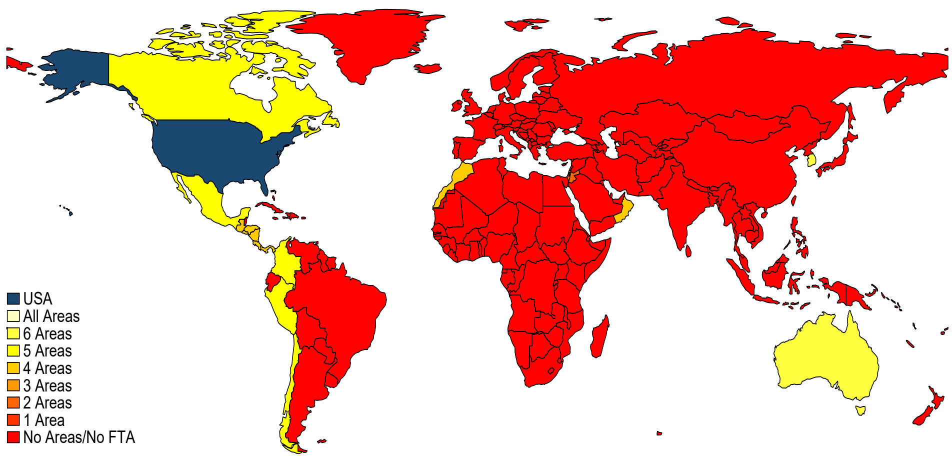
In Figure B.7, we represent bilateral regulatory distances from Costa Rica (Figure B.7a), the US (Figure B.7b), France (Figure B.7c) and China (Figure B.7d) to any other country of the world.

Figure B.7: Bilateral Regulatory Distances From Certain Origin Countries

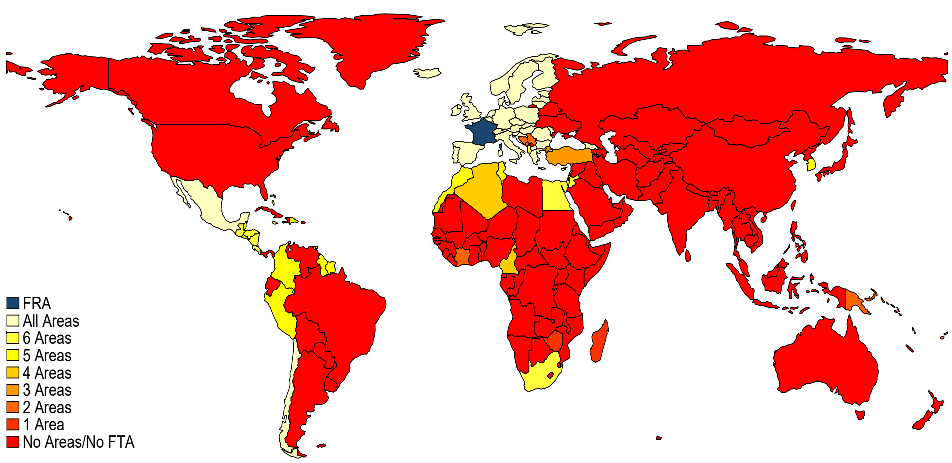
(a) From Costa Rica



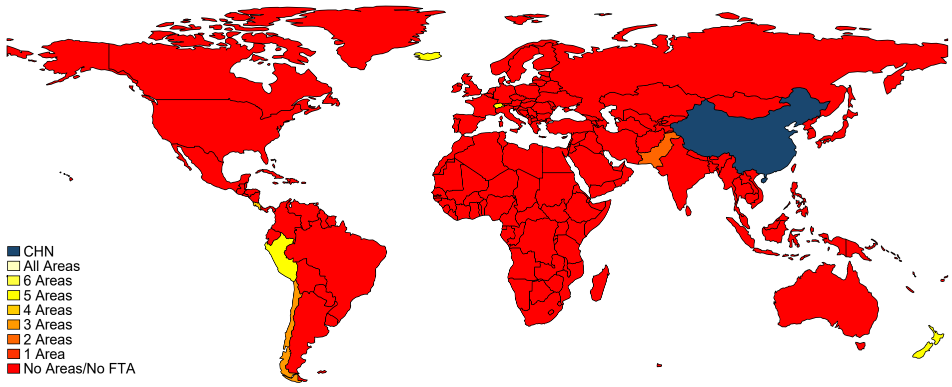
(b) From the United States



(c) From France



(d) From China



Notes: Each of the four panels in this figure illustrate the countries with which Costa Rica (in panel (a)), the United States (in panel (b)), France (in panel (c)), and China (in panel (d)) share in 2015 a PTA containing provisions in at least one of the seven policy areas listed in footnote 10. If it does, it indicates in how many of the seven areas the corresponding PTA contains some provision.

Panel (a) shows that Costa Rica has deep integration agreements with Canada, members of the European Common Market, Panama, the Dominican Republic, and Peru, and more shallow agreements with China, Chile, and other Central and North American countries. Panel (b) shows the US has a deep PTA with Canada and Mexico (NAFTA), as well as with Colombia, Peru, Chile and Australia (these four are bilateral trade agreements), and a more shallow agreement with Central American countries (CAFTA). In the case of France, panel (c) illustrates that it has deep PTAs not only with the other members of the European Common Market, but also with countries in North America (Mexico), Central America (e.g., Guatemala, Honduras, or Costa Rica), South America (e.g., Colombia, Peru, or Chile), Africa (e.g., Morocco, Tunisia, Egypt, or South Africa), and Asia (South Korea). Panel (d) shows that China has deep trade integration agreements with comparatively few and smaller countries (e.g., Iceland, Switzerland, Peru, or New Zealand).

In sum, Figure B.7 shows that countries differ in the number and identity of the potential trade partners with whom they have signed deep PTAs. Furthermore, it is common for countries to sign deep PTAs with other countries that are neither geographically nor linguistically close to them (e.g., Costa Rica and China, the US and South Korea, or France and South Africa).

B.5 Correlation in Export Choices: Additional Results

To generate the variable Y^g_{ijt} used to compute the results in Section 2.2, we set $\bar{n}_g$ so that we classify two countries as geographically close if their distance is less than 790 km, which is the 2.5 percentile of the distribution of distances across all country pairs. According to this threshold, e.g., Spain and Portugal, and France and Germany, are geographically close. Conversely, e.g., Spain and Austria, or France and Denmark, are not. Analogously, we classify two countries as linguistically close if the probability two randomly selected individuals from both countries speak a common language is at least 0.89 (i.e., if $n^l_{jj'} < 0.11$, where 0.11 is the 2.5 percentile of the distribution of linguistic distances across all country pairs), and we classify two countries as regulatory close if they are cosignatories of a PTA including provisions in at least four of the seven areas listed in footnote 10 (i.e., if $n^a_{jj't} < 0.43$). According to these thresholds, e.g., Argentina and Spain (but not France and Switzerland) are linguistically close; and all members of the EU, NAFTA, CAFTA, or Mercosur are regulatory close to each other.

Table B.4: Conditional Export Probabilities

	Panel A: No Controls				*Panel B: Controlling for Firm-Year Fixed Effects*			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Y_{ijt}^{g}	0.1904^{a} (0.0072)			0.1345^{a} (0.0059)	0.1529^{a} (0.0068)			0.1217^{a} (0.0060)
Y_{ijt}^{l}		0.1334^{a} (0.0057)		0.0733^{a} (0.0038)		0.1091^{a} (0.0050)		0.0760^{a} (0.0041)
Y_{ijt}^{a}			0.0825^{a} (0.0037)	0.0297^{a} (0.0016)			0.0517^{a} (0.0026)	0.0222^{a} (0.0018)
Obs.	3,859,618				3,859,618			
	Panel C: Controlling for Sector-Country-Year Fixed Effects				*Panel D: Controlling for Sector-Country-Year & Firm-Year Fixed Effects*			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Y_{ijt}^{g}	0.1785^{a} (0.0069)			0.1269^{a} (0.0057)	0.1384^{a} (0.0065)			0.1116^{a} (0.0057)
Y_{ijt}^{l}		0.1277^{a} (0.0054)		0.0706^{a} (0.0035)		0.1013^{a} (0.0048)		0.0721^{a} (0.0039)
Y_{ijt}^{a}			0.0779^{a} (0.0035)	0.0283^{a} (0.0015)			0.0431^{a} (0.0025)	0.0169^{a} (0.0017)
Obs.	3,859,618				3,859,618			

Note: [a] denotes 1% significance. Standard errors clustered by firm. The dependent variable in all specifications is a dummy that equals one if firm i exports to country j in year t. The covariates are $Y_{ijt}^{x} = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n_{jj'}^{x} \leqslant \bar{n}_x\} y_{ij't} > 0\}$ for $x \in \{g, l\}$, and $Y_{ijt}^{a} = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n_{jj't}^{a} \leqslant \bar{n}_a\} y_{ij't} > 0\}$, with $\bar{n}_g = 1.153$, $\bar{n}_l = 0.5$ and $\bar{n}_a = 0.78$.

In Table B.4, we present estimates analogous to those in Table 1 for alternative threshold values $\bar{n}_g$, $\bar{n}_l$, and $\bar{n}_a$. Here, we set $\bar{n}_g = 1.153$ (or 1,153 km), $\bar{n}_l = 0.5$ and $\bar{n}_a = 0.78$. The values of $\bar{n}_g$ and $\bar{n}_l$ we use here equal the 5th percentile of the distribution of the corresponding distance measure between any pair of countries in our sample; the value $\bar{n}_a = 0.72$ is equivalent to characterizing as deep any PTA that contains a provision in at least two of the seven areas listed in footnote 10.

A comparison of the estimates in tables 1 and B.4 reveals that, as we increase the set of countries classified as being geographically or linguistically close to a destination j, or as being cosignatories of a deep PTA with j, the impact that exporting to at least one of these countries has on the probability of exporting to j decreases. For example, comparing the estimate of the coefficient on Y_{ijt}^{g} in column (4) of Panel D in Table 1 to that in Table B.4, we observe that the difference in the predicted export probability to any given destination is 0.18 when comparing firms that export to at least one country that is less than 790 km away from it to those that do not, but only 0.11 when comparing firms that export to at least one country that is less than $1,153$ km away from it to those that do not. This is consistent with the correlation in a firm's export participation decisions in any two countries decreasing in the geographical distance between both countries.

In Table B.5, we present estimates analogous to those in Table 1, but adding the lagged dependent variable as an additional covariate. We observe that the point estimate of the coefficient on the lagged dependent variable, y_{ijt-1}, is always statistically different from zero and economically

Table B.5: Conditional Export Probabilities with Lagged Export Participation

	Panel A: No Controls				*Panel B: Controlling for Firm-Year Fixed Effects*			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
y_{ijt-1}	0.6529^a	0.6821^a	0.6809^a	0.6411^a	0.6469^a	0.6697^a	0.6754^a	0.6411^a
	(0.0090)	(0.0090)	(0.0091)	(0.0088)	(0.0089)	(0.0087)	(0.0089)	(0.0086)
Y^g_{ijt}	0.0990^a			0.0815^a	0.0834^a			0.0754^a
	(0.0036)			(0.0034)	(0.0037)			(0.0035)
Y^l_{ijt}		0.0574^a		0.0303^a		0.0430^a		0.0278^a
		(0.0024)		(0.0022)		(0.0023)		(0.0023)
Y^a_{ijt}			0.0300^a	0.0144^a			0.0162^a	0.0066^a
			(0.0011)	(0.0008)			(0.0009)	(0.0008)
Obs.	3,353,236				3,353,236			

	Panel C: Controlling for Sector-Country-Year Fixed Effects				*Panel D: Controlling for Sector-Country-Year & Firm-Year Fixed Effects*			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
y_{ijt-1}	0.6387^a	0.6638^a	0.6634^a	0.6411^a	0.6319^a	0.6506^a	0.6567^a	0.6269^a
	(0.0089)	(0.0089)	(0.0089)	(0.0088)	(0.0086)	(0.0086)	(0.0088)	(0.0085)
Y^g_{ijt}	0.0980^a			0.0797^a	0.0800^a			0.0726^a
	(0.0037)			(0.0034)	(0.0037)			(0.0035)
Y^l_{ijt}		0.0594^a		0.0322^a		0.0434^a		0.0290^a
		(0.0026)		(0.0023)		(0.0025)		(0.0024)
Y^a_{ijt}			0.0298^a	0.0142^a			0.0140^a	0.0048^a
			(0.0011)	(0.0008)			(0.0010)	(0.0008)
Obs.	3,353,236				3,353,236			

Note: ^{a} denotes 1% significance. Standard errors are clustered by firm. The dependent variable is a dummy that equals 1 if firm i exports to country j in year t, y_{ijt}. The covariates of interest are the lagged dependent variable, y_{ijt-1}, $Y^x_{ijt} = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n^x_{jj'} \leqslant \bar{n}_x\} y_{ij't} > 0\}$ for $x \in \{g, l\}$, and $Y^a_{ijt} = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n^a_{jj't} \leqslant \bar{n}_a\} y_{ij't} > 0\}$, with $\bar{n}_g = 790$ km, $\bar{n}_l = 0.11$ and $\bar{n}_a = 0.43$.

important. Holding everything else constant, the probability a firm exports to a destination in a given year is approximately 65% larger if the firm also exported to the same destination in the previous year. A comparison of the estimates of the coefficients on Y^g_{ijt}, Y^l_{ijt}, and Y^a_{ijt}, in tables 1 and B.5 shows that these estimates decrease when we control for the lagged dependent variable. However, they all remain statistically significant at the 5% significance level.

C Equation for Potential Export Revenues: Details

We derive the expression in equation (5) in three steps.

First Step. Assuming firm i's marginal cost of selling in the home market h at period t is $\tau_{ht} w_{it}$, and that the firm also faces at home a CES demand function with demand elasticity η, the revenue firm i obtains in h at t is

$$r_{iht} = \left[\frac{\eta}{\eta - 1} \frac{\tau_{ht} w_{it}}{P_{ht}}\right]^{1-\eta} Y_{ht}. \tag{C.1}$$

Combining equations (3) and (C.1), we rewrite the potential export revenues of firm i in country j at period t as a function of its revenue in the domestic market:

$$r_{ijt} = \left[\frac{\tau_{ijt}}{\tau_{ht}} \frac{P_{ht}}{P_{jt}}\right]^{1-\eta} \frac{Y_{jt}}{Y_{ht}} r_{iht}. \tag{C.2}$$

Second Step. Substituting $(\tau_{ijt})^{1-\eta}$ in equation (C.2) by its expression in equation (4), we obtain

$$r_{ijt} = \exp(\xi_y y_{ijt-1} + \check{\xi}_{jt} + \alpha_s + \alpha_a \ln(a_{sjt}) + \xi_w \ln(w_{it}) + \ln(r_{iht})), \tag{C.3}$$

with

$$\check{\xi}_{jt} = \xi_{jt} + (1 - \eta) \ln(P_{ht}/P_{jt}) + \ln(Y_{jt}/Y_{ht}) - (1 - \eta) \ln(\tau_{ht}). \tag{C.4}$$

Third Step. Taking the logarithm of both sides of equation (C.1) and rearranging terms, we obtain

$$\ln(w_{it}) = \frac{1}{1 - \eta}(\ln(r_{iht}) - \ln(Y_{ht})) + \ln(\eta - 1) - \ln(\eta) + \ln(P_{ht}) - \ln(\tau_{ht}).$$

Plugging this equality into equation (C.3), we obtain equation (5) with $\alpha_s = \xi_s$, $\alpha_a = \xi_a$, and

$$\alpha_{jt} = \check{\xi}_{jt} + \xi_w(-(1/(1 - \eta)) \ln(Y_{ht}) + \ln(\eta - 1) - \ln(\eta) + \ln(P_{ht}) - \ln(\tau_{ht})), \tag{C.5a}$$
$$\alpha_r = 1 + \xi_w/(1 - \eta). \tag{C.5b}$$

D Solution Algorithm: Additional Details

D.1 Additional Steps

Step 2. Denote by τ the smallest t with $\check{\bar{y}}_{it} > \check{\underline{y}}_{it}$. In this step, we tighten our bounds at τ. The procedure differs from that in step 1 in that we now solve the problem in equation (21) only for $t = \tau$ at the state $(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$, and we do so using constant upper bounds that condition on this state. The new initial constant upper bounds equal the firm's choices implied by the upper-bound policies $\{\bar{o}^*_{it}(\cdot)\}_{t \geqslant \underline{t}_i}$ when the state at τ is $(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$ and the blocking shocks for all $t > \tau$ equal the smallest value in their support. Formally, the new initial constant upper bound for period τ is $\bar{o}^*_{i\tau}(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$, and, for all $t > \tau$, we compute these through the following iterative procedure

$$\bar{b}^{[0]}_{it'|\tau} = \bar{o}^*_{it'}(\bar{b}^{[0]}_{it'-1|\tau}, \underline{\omega}_J), \quad \text{for } t' = \tau+1, \ldots, t, \text{ with initial value } \bar{b}^{[0]}_{i\tau|\tau} = \bar{o}^*_{i\tau}(\check{y}_{i\tau-1}, \check{\omega}_{i\tau}). \tag{D.1}$$

Solving the problem in equation (21) with these new constant upper bounds, we obtain new upper-bound policies for all $t \geqslant \tau$. As in step 1, we use these policies and a procedure analogous to that in equation (D.1) to compute new constant upper bounds, which we use to solve again the problem in equation (21) and obtain in this way new upper-bound policies. We implement this procedure until convergence (see Appendix A), denoting as $\bar{o}^*_{it|\tau}(\cdot)$ the resulting upper-bound policy for any $t \geqslant \tau$. We use these policies, in combination with similarly computed lower-bound policies $\underline{o}^*_{it|\tau}(\cdot)$, to obtain bounds on the firm's optimal choice at period τ at the path of interest:

$$\check{\bar{y}}_{i\tau|\tau} = \bar{o}^*_{i\tau|\tau}(\check{y}_{i\tau-1}, \check{\omega}_{i\tau}), \qquad \text{and} \qquad \check{\underline{y}}_{i\tau|\tau} = \underline{o}^*_{i\tau|\tau}(\check{y}_{i\tau-1}, \check{\omega}_{i\tau}). \tag{D.2}$$

If these bounds coincide, they equal the optimal choice at τ at $(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$. If so, we proceed to the next period τ' at which the bounds computed in step 1 differ, implementing again the step 2 procedure to tighten the bounds at τ'. If the bounds in equation (D.2) differ, we move to step 3.

Step 3. In this step, we tighten further the bounds at τ. For every country j for which the bounds in equation (D.2) do not coincide, we solve a problem that differs from that in equation (21) in that, for period $\tau+1$ and a subset of countries M that does not include j, we condition on *functional* (instead of constant) upper bounds. Specifically, for any j such that $\check{\bar{y}}_{ij\tau|\tau} > \check{\underline{y}}_{ij\tau|\tau}$, we compute

$$\max_{y_{ij\tau}} \Big\{ y_{ij\tau}(u_{ij\tau}(\check{y}_{ij\tau-1}, \check{\omega}_{ij\tau}) + \sum_{j' \neq j} \check{\bar{y}}_{ij'\tau|\tau}(c_{jj't} + c_{j'jt})) + \delta \mathbb{E}_{i\tau} \tilde{V}_{ij\tau+1}(y_{ij\tau}, \omega_{ij\tau+1}, \{\omega_{ij'\tau+1}\}_{j' \in M}) \Big\}, \tag{D.3}$$

with

$$\tilde{V}_{ij\tau+1}(y_{ij\tau}, \omega_{ij\tau+1}, \{\omega_{ij'\tau+1}\}_{j'\in M}) = \max_{y_{ij\tau+1}} \Big\{ y_{ij\tau+1}(u_{ij\tau+1}(y_{ij\tau}, \omega_{ij\tau+1}) + \delta \mathbb{E}_{i\tau+1} \bar{V}_{ijt\tau+2}(y_{ij\tau+1}, \omega_{ij\tau+2}) + \sum_{j' \in M} \bar{b}_{ij'\tau+1|\tau}(\omega_{ij'\tau+1})(c_{jj'\tau+1} + c_{j'j\tau+1}) + \sum_{j' \notin M} \mathbb{1}\{j' \neq j\} \bar{b}^*_{ij'\tau+1|\tau}(c_{jj'\tau+1} + c_{j'j\tau+1})) \Big\}. \tag{D.4}$$

The function $\tilde{V}_{ijt\tau+2}(y_{ij\tau+1}, \omega_{ij\tau+2})$ is country j's value function when the firm's choice in every period $t \geqslant \tau+2$ and every country other than j is set to the constant upper bounds obtained in the last iteration of the step 2 procedure. For every country j' other than j, equation (D.4) imposes the upper bounds

$$\bar{b}_{ij'\tau+1|\tau}(\omega_{ij'\tau+1}) = \bar{o}^*_{ij'\tau+1|\tau}(\check{\bar{y}}_{ij'\tau|\tau}, \omega_{ij'\tau+1}), \qquad \text{if } j' \in M, \tag{D.5a}$$

$$\bar{b}^*_{ij'\tau+1|\tau} = \bar{o}^*_{ij'\tau+1|\tau}(\check{\bar{y}}_{ij'\tau|\tau}, \underline{\omega}), \qquad \text{if } j' \notin M, \tag{D.5b}$$

where $\bar{o}^*_{ij'\tau+1|\tau}(\cdot)$ and $\check{\bar{y}}_{ij'\tau|\tau}$ are computed in step 2. By definition, $\bar{b}_{ij'\tau+1|\tau}(\omega_{ij'\tau+1}) \leqslant \bar{b}^*_{ij'\tau+1|\tau}$. Thus, the bounds computed in step 3 are tighter than those computed in step 2, and they will be tighter the larger the set M. However, solving the problem in equation (D.3) requires computing an expectation over the vector $(\omega_{ij\tau+1}, \{\omega_{ij'\tau+1}\}_{j'\in M})$, a step that is computationally more complicated the larger the set M. In our application, for a country j, we choose M as the 16 countries that are geographically closer to j. If the step 3 upper and lower bounds do not coincide at $(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$ at τ, we proceed to step 4.

Step 4. In this step, we tighten further the bounds at period τ. To do so, we solve an optimization problem that differs from those solved in steps 1 to 3 in that, instead of computing policy functions iteratively country by country, we do so for several countries simultaneously.

Consider a set M of countries for which the step 3 upper and lower bounds on the firm's optimal choices at the path of interest do not coincide at τ. For any $t \geqslant \tau$, define vectors y_{iMt} and ω_{iMt} that, for t and all $j \in M$, include the choice y_{ijt} and blocking shock ω_{ijt}, respectively. Define also

$$\bar{V}_{iM\tau+h}(y_{iM\tau+h-1}, \omega_{iM\tau+h}) = \sum_{j\in M} \bar{V}_{ij\tau+h}(y_{ij\tau+h-1}, \omega_{ij\tau+h}), \tag{D.6}$$

where $\bar{V}_{ij\tau+h}(\cdot)$ is the country j's value function that results from equating the firm's choice in all periods $t \geqslant \tau + h$ and all countries other than j to the constant upper bounds obtained in the last iteration of the step 2 procedure. In step 4, we solve for all $t \in [\tau, \tau + h - 1]$ the problem

$$\begin{aligned}\bar{V}_{iMt}(y_{iMt-1}, \omega_{iMt}) = \max_{y_{iMt}\in\{0,1\}^M} \Big\{ \sum_{j\in M} \big\{ y_{ijt}(u_{ijt}(y_{ijt-1}, \omega_{ijt}) + \sum_{j'\in M} y_{ij't}c_{jj't} + \\ \sum_{j'\notin M} \mathbb{1}\{j' \neq j\}\bar{b}^*_{ij'\tau|\tau}(c_{jj't} + c_{j'jt}))\big\} + \delta\mathbb{E}_{it}\bar{V}_{iMt+1}\left(y_{iMt}, \omega_{iMt+1}\right) \Big\},\end{aligned} \tag{D.7}$$

with $\bar{b}^*_{ij'\tau|\tau}$ and $\bar{V}_{iM\tau+h}(\cdot)$ defined as in equations (D.5b) and (D.6), respectively. Solving this problem is computationally more complicated the larger the set M and the horizon h are. If there are less than ten countries for which the step 3 bounds at the state of interest at period τ differ, we include them all in M. If there are more than ten countries for which the step 3 bounds differ, we solve the problem in equation (D.7) repeatedly for different sets of countries, grouping together in these sets those countries that are geographically close to each other. Concerning h, we solve first the problem for $h = 1$, and increase progressively its value until $h = 10$.

Step 5. In this step, we tighten further the bounds at τ. We compute the firm's optimal export paths in a set M of countries fixing the firm's choices in all countries not in M to constant upper bounds. Specifically, we first solve the following period-T problem for every value of $(y_{iMT-1}, \omega_{iMT})$:

$$\begin{aligned}\bar{V}_{iMT}(y_{iMT-1}, \omega_{iMT}) = \max_{y_{iMT}\in\{0,1\}^M} \Big\{ \sum_{j\in M} \big\{ y_{ijT}(u_{ijT}(y_{ijT-1}, \omega_{ijT}) + \sum_{j'\in M} y_{ij'T}c_{jj'T} + \\ \sum_{j'\notin M} \mathbb{1}\{j \neq j'\}\bar{b}^*_{ij'T|\tau}(c_{jj'T} + c_{j'jT}))\big\} + \delta\mathbb{E}_{iT}\bar{V}_{iMT+1}\left(y_{iMT}, \omega_{iMT+1}\right) \Big\}.\end{aligned} \tag{D.8}$$

As this problem is stationary, we use value-function iteration to solve for the value function $\bar{V}_{iMT}(\cdot)$. Given $\bar{V}_{iMT}(\cdot)$, we use backward induction to solve for the optimal policy in M for all $t \in [\tau, T]$.

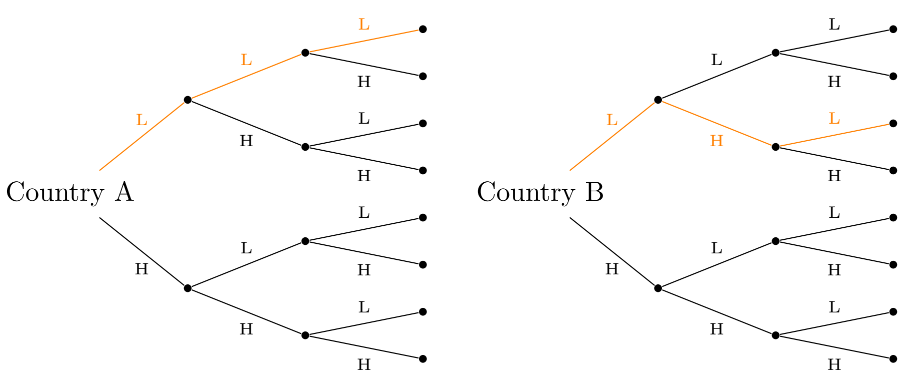
If M includes all J foreign countries, the problem in equation (D.8) coincides with that in equation (18) and, thus, its solution yields the firm's optimal policy function. Solving the problem in equation (18) for a large set M is computationally infeasible. In our application, we choose M according to the following rules. If there are less than six countries for which step 4 upper and lower bounds on the optimal choice at the path of interest at period τ differ, we include them all in M. If there are more than six countries for which the step 4 bounds differ, we implement the step 5 algorithm repeatedly for different sets of six countries grouping together countries that are geographically close to each other.

Closing the algorithm. If there are countries for which the upper and lower bound on the optimal choice at the path of interest at period τ differ after step 5, we assume the optimal choice is to not export to those countries at τ at the state of interest.

D.2 Algorithm in a Two-Country and Three-Period Setting

We illustrate the mechanics of our algorithm in an example with two countries (A and B) and three periods. We use trees to represent graphically all possible paths of ω_{ijt}. With the letters L (with stands for low) and H (which stands for high), we denote the events in which the blocking shock respectively equals the smallest, $\underline{\omega}$, and largest, $\bar{\omega}$, values in their support. E.g., in Figure D.1, the orange path is one in which blocking shocks in A are low in all three periods while, in B, these are low in periods 1 and 3, and high in period 2.

Figure D.1: Possible Paths of Fixed Cost Shocks

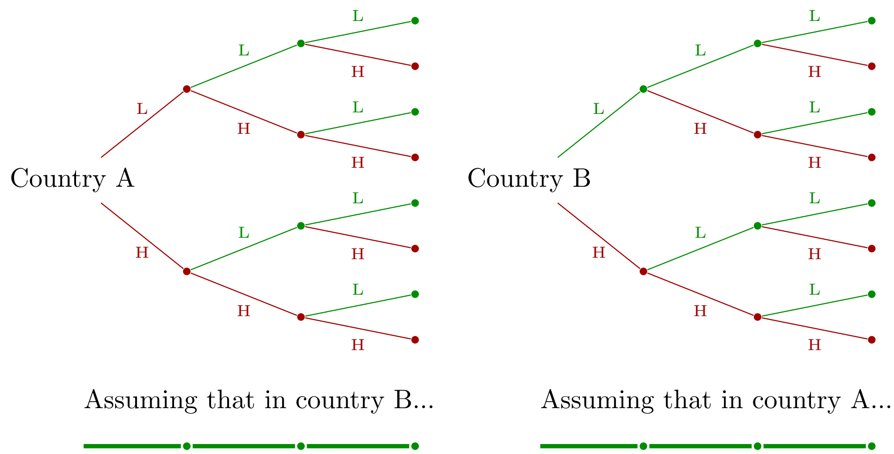


Step 1. In Figure D.2, we illustrate the first iteration of step 1 of the algorithm (see Section 4). The left panel illustrates the solution to the optimization problem in equation (21) for country A when setting $\bar{b}_{iBt} = 1$ for all three time periods; the right panel is analogous but for country B. Using the notation in Section 4, Figure D.2 thus illustrates the upper-bound policy function

$$\bar{o}^{[0]}_{it}(y_{it-1}, \omega_{it}) = (\bar{o}^{[0]}_{iAt}(y_{iAt-1}, \omega_{iAt}), \bar{o}^{[0]}_{iBt}(y_{iBt-1}, \omega_{iBt})), \qquad \text{for all } t = \{1, 2, 3\}. \tag{D.9}$$

In all figures in this section, we use green to identify branches at which the firm exports, and red to identify branches at which it does not. The left panel in Figure D.2 thus shows that, conditional on the firm exporting to B in all periods and states (as reflected by the three green segments under "Assuming that in country B..."), the firm chooses not to export to A at $t = 1$

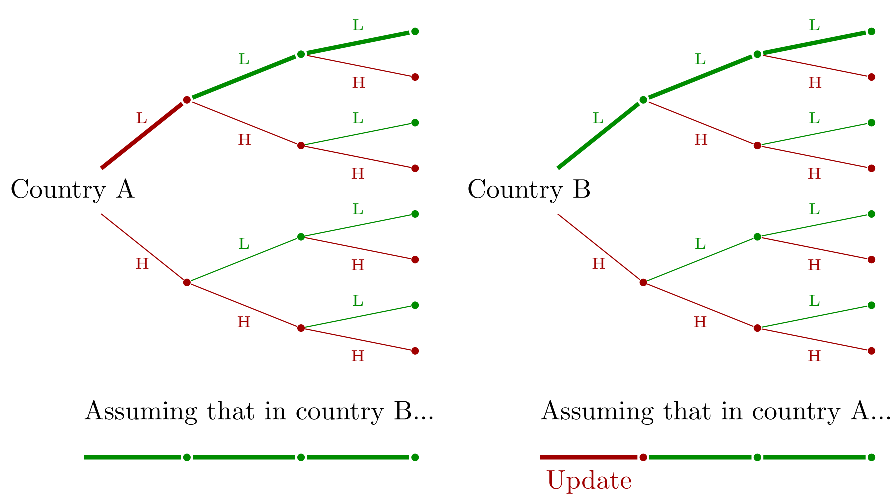
Figure D.2: Initial Upper-Bound Policy Functions



regardless of whether ω_{iA1} is high or low (as reflected by the two red segments branching out from the "Country A" label), and chooses to export to A at $t = 2$ and $t = 3$ if and only if ω_{iAt} in the corresponding period t is low (as reflected by the L-segments being green and the H-segments being red). Similarly, the right panel in Figure D.2 shows that, if the firm exports to A in all periods and states (as reflected by the three green segments under "Assuming that in country A...\"), the firm chooses to export to B in any given period if and only if ω_{iBt} in the corresponding period t is low (as reflected by the L-segments being green and the H-segments being red).

In Figure D.3, we evaluate the upper-bound policy in equation (D.9), as represented in Figure D.2, at the path of shocks in which these equal their lowest possible value in every country and period (i.e., the path marked by thick lines in each tree's top branch). Doing so, we obtain new constant upper bounds on the firm's choice in all countries and periods. E.g., as the upper-bound policy represented in Figure D.2 prescribes the firm not to export to A at $t = 1$ even $\omega_{iA1} = \underline{\omega}$, we update from one to zero the constant upper bound in A at $t = 1$ (as reflected in the change in

Figure D.3: Updated Constant Upper Bounds

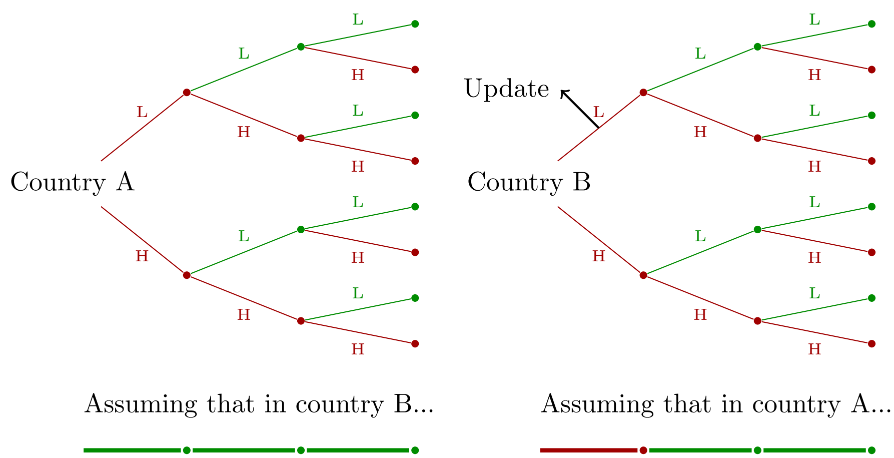


color of the segment labeled "Update"). Using the notation in Section 4, it is thus the case that

$$(\bar{b}^{[1]}_{iA1}, \bar{b}^{[1]}_{iA2}, \bar{b}^{[1]}_{iA3}) = (0, 1, 1) \qquad \text{and} \qquad (\bar{b}^{[1]}_{iB1}, \bar{b}^{[1]}_{iB2}, \bar{b}^{[1]}_{iB3}) = (1, 1, 1). \tag{D.10}$$

We represent in Figure D.4 the new upper-bound policy function we obtain by solving again the optimization problem in equation (21) but now conditioning on the constant upper bounds illustrated at the bottom of Figure D.4, and listed in equation (D.10). Comparing figures D.2 and D.4, we observe that the change in the constant upper bound in country A at period $t = 1$ drives a change in the upper-bound policy function in country B at $t = 1$ at the low fixed cost shock segment, whose color switches from green to red. As country B's constant upper bounds in figures D.2 and D.4 coincide, the upper-bound policy function in country A remains the same.

Figure D.4: Updated Upper-Bound Policy Functions



In Figure D.5, we evaluate the updated upper-bound policy illustrated in Figure D.4 at the path of shocks at which these equal their lowest possible value in every country and period, represented in Figure D.3 by the thick lines in each tree's top branch. Comparing figures D.3 and D.5, we observe that the update in the upper-bound policy in Figure D.4 with respect to that in Figure D.2 allows to update from one to zero the constant upper bound in B at $t = 1$ (as reflected in the change in color of the segment labeled "Update"). It is then the case that

$$(\bar{b}^{[2]}_{iA1}, \bar{b}^{[2]}_{iA2}, \bar{b}^{[2]}_{iA3}) = (0, 1, 1) \qquad \text{and} \qquad (\bar{b}^{[2]}_{iB1}, \bar{b}^{[2]}_{iB2}, \bar{b}^{[2]}_{iB3}) = (0, 1, 1). \tag{D.11}$$

Continuing with the iterative process, we solve again the optimization problem in equation (21) but now conditioning on the updated constant upper bounds illustrated at the bottom of Figure D.5 and listed in equation (D.11). The solution is an upper-bound policy identical to that obtained in the previous iteration; i.e., that in Figure D.4. Intuitively, as the upper-bound policy in Figure D.4 already prescribes the firm not to export to A at $t = 1$, regardless of the value of ω_{iA1}, the update in the constant upper bound in B at $t = 1$ does not change the upper-bound policy function in A. Thus, after two iterations, the upper-bound policy function has converged. We represent this upper-bound policy in Figure D.6.

We follow analogous steps to compute lower-bound policy functions. Assume for simplicity the converged lower-bound policies prescribe the firm not to export to any country in any period regardless of the value of ω_{ijt} for any j and t. The converged lower-bound policy thus corresponds to that in Figure D.7.

Figure D.5: Updated Constant Upper Bounds

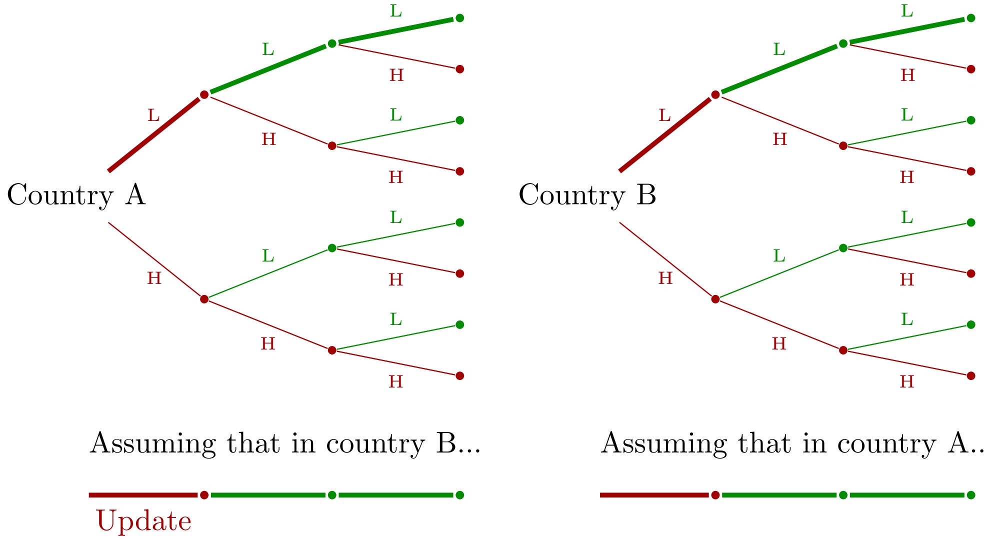


Figure D.6: Upper-Bound Policy Functions After Convergence

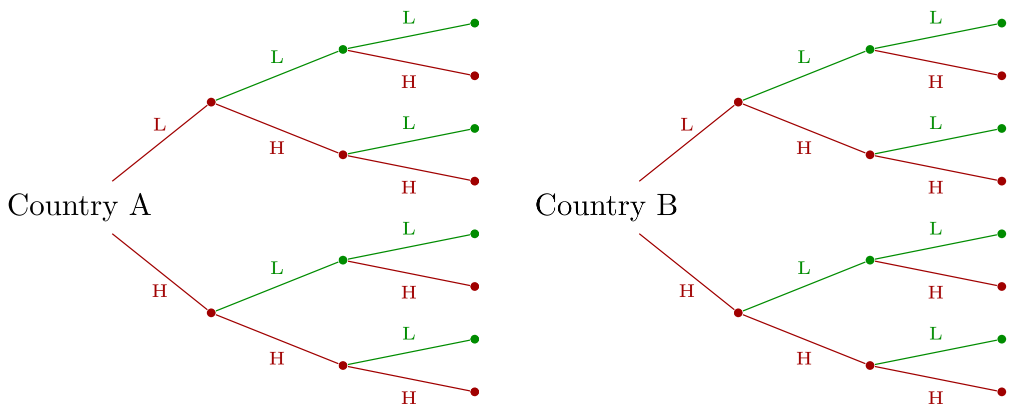


Figure D.7: Lower-Bound Policy Functions After Convergence

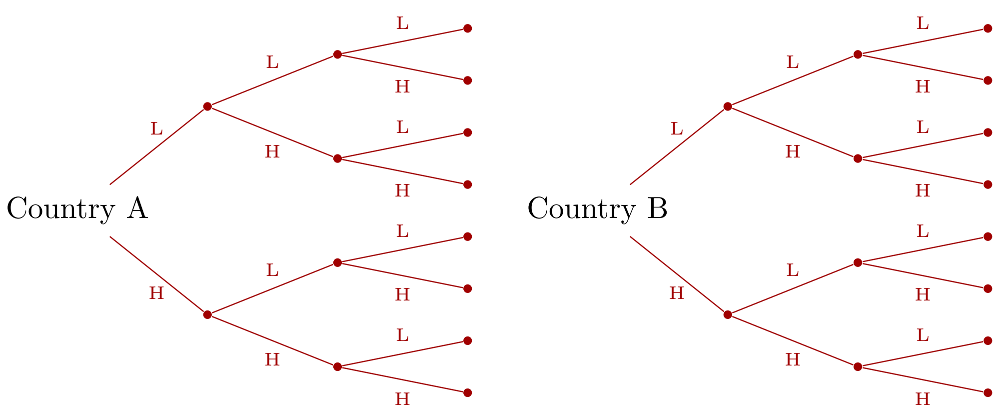
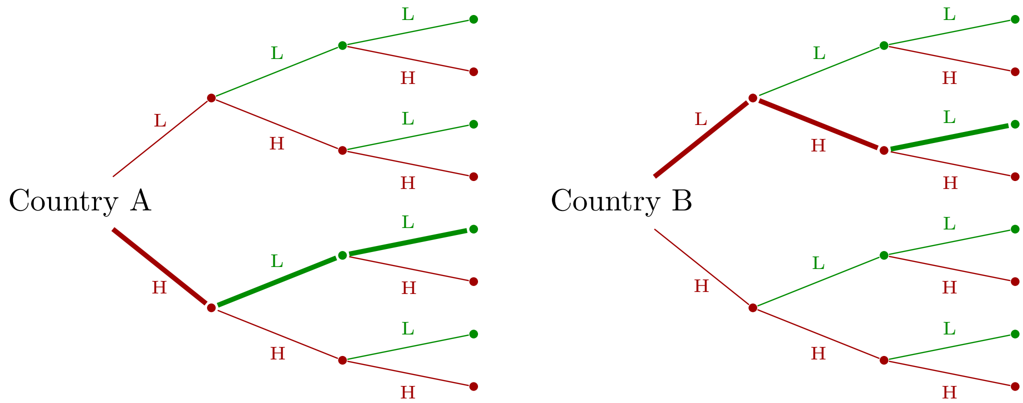


Figure D.8: Evaluating Upper-Bound Policy Functions at Path of Interest



The final stage in step 1 of our algorithm is to evaluate the converged lower- and upper-bound policy functions at a specific path of interest. Assume, e.g., this path is:

$$(\hat{\omega}_{iA1}, \hat{\omega}_{iA2}, \hat{\omega}_{iA3}) = (\bar{\omega}, \underline{\omega}, \underline{\omega}) \qquad \text{and} \qquad (\hat{\omega}_{iB1}, \hat{\omega}_{iB2}, \hat{\omega}_{iB3}) = (\underline{\omega}, \bar{\omega}, \underline{\omega}), \tag{D.12}$$

where, as a reminder, $\underline{\omega}$ and $\bar{\omega}$ are represented by L and H, respectively, in all figures.

Figure D.8 is identical to Figure D.6 except that the path of interest is highlighted. The colors of the highlighted branches indicate the upper bounds on the firm's optimal choices at the path of interest; i.e.,

$$(\bar{\check{y}}_{iA1}, \bar{\check{y}}_{iA2}, \bar{\check{y}}_{iA3}) = (0, 1, 1) \qquad \text{and} \qquad (\bar{\check{y}}_{iB1}, \bar{\check{y}}_{iB2}, \bar{\check{y}}_{iB3}) = (0, 0, 1). \tag{D.13}$$

Similarly, given the converged lower-bound policy function in Figure D.7, the lower bounds on the firm's optimal choices at the path of interest are

$$(\underline{\check{y}}_{iA1}, \underline{\check{y}}_{iA2}, \underline{\check{y}}_{iA3}) = (0, 0, 0) \qquad \text{and} \qquad (\underline{\check{y}}_{iB1}, \underline{\check{y}}_{iB2}, \underline{\check{y}}_{iB3}) = (0, 0, 0). \tag{D.14}$$

The bounds coincide at $t = 1$ for both countries; thus, the optimal choices at $t = 1$ at the path of interest are $(\check{y}_{iA1}, \check{y}_{iB1}) = (0, 0)$. At $t = 2$, both bounds differ in their prescribed choice in A.

Step 2. In this step, we tighten the bounds at $t = 2$. To do so, we first compute new constant upper bounds that condition on the state of interest at $t = 2$; i.e., we evaluate the policy function in Figure D.6 along a path that, for $j = \{A, B\}$, sets $\omega_{ijt} = \breve{\omega}_{ijt}$ for $t \leqslant 2$, and $\omega_{ijt} = \underline{\omega}$ for $t > 2$. In Figure D.9, we recover the upper-bound policy in Figure D.6, fade all branches that cannot be reached from the state of interest at $t = 2$ and mark with a wide line the relevant path. Conditioning on the state of interest up to $t = 2$ permits updating the constant upper bound in B at $t = 2$ (as reflected in the change in color of the segment labeled "Update" in Figure D.9). Using the notation in Section 4, it then holds that

$$(\bar{b}^{[0]}_{iA2|2}, \bar{b}^{[0]}_{iA3|2}) = (1, 1) \qquad \text{and} \qquad (\bar{b}^{[0]}_{iB2|2}, \bar{b}^{[0]}_{iB3|2}) = (0, 1). \tag{D.15}$$

We represent in Figure D.10 the upper-bound policy function obtained by solving the optimization problem in equation (21) for $t \geqslant 2$ with the new constant upper bounds represented at the bottom of Figure D.9 and listed in equation (D.15). Figure D.10 shows that the upper-bound policy in A at $t = 2$ is updated.

Figure D.9: Initial Constant Upper Bounds That Condition on Path of Interest for $t \leqslant 2$

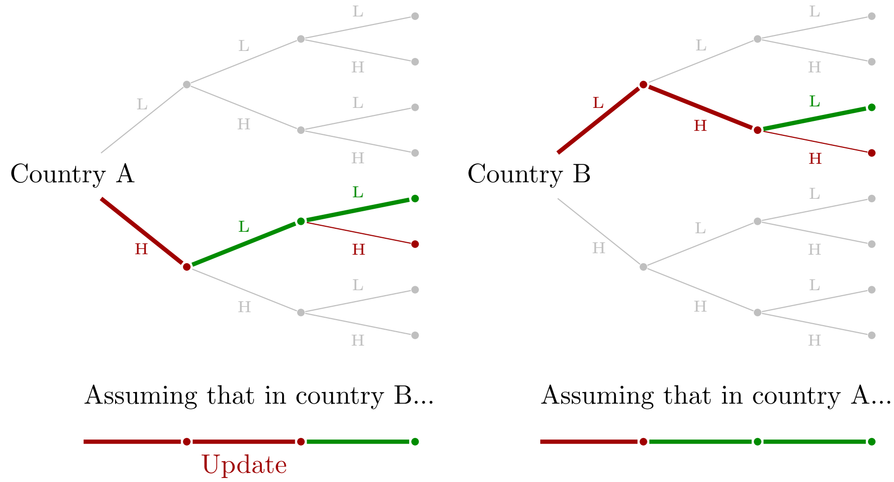
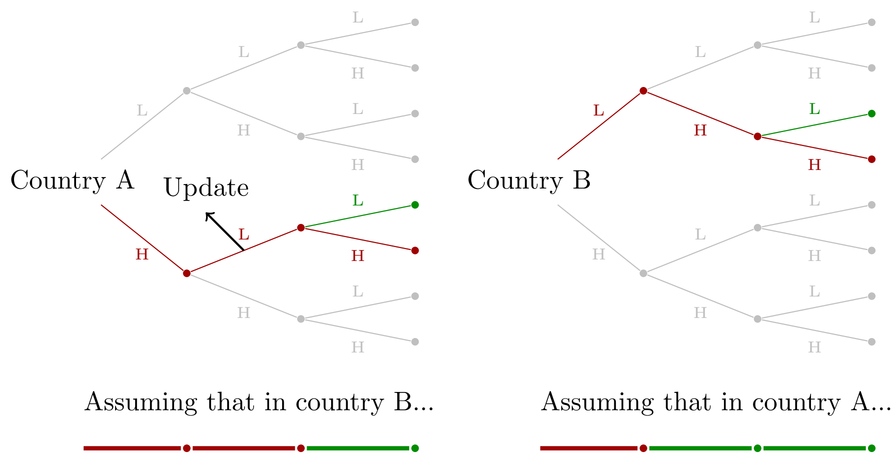


Figure D.10: Upper-Bound Policy Functions That Condition on Path of Interest for $t \leqslant 2$



Next, we evaluate the updated upper-bound policies in Figure D.10 along the path that, for $j = \{A, B\}$, sets $\omega_{ijt} = \breve{\omega}_{ijt}$ for $t \leqslant 2$ and $\omega_{ijt} = \underset{\sim}{\omega}$ for $t > 2$, represented in Figure D.11 by thick lines. Comparing figures D.9 and D.11, we observe that the update in the upper-bound policy in Figure D.10 relative to that in Figure D.8 allows us to update the constant upper bound in A at $t = 2$ (see the red segment over the label "Update" in Figure D.11). It is then the case that

$$(\bar{b}^{[1]}_{iA2|2}, \bar{b}^{[1]}_{iA3|2}) = (0, 1) \qquad \text{and} \qquad (\bar{b}^{[1]}_{iB2|2}, \bar{b}^{[1]}_{iB3|2}) = (0, 1). \tag{D.16}$$

Continuing with this iterative procedure, we solve again the problem in equation (21) for periods $t \geqslant 2$, but now conditioning on the new constant upper bounds in equation (D.16) (see also bottom of Figure D.11). The solution to this problem yields upper-bound policy functions identical to those obtained in the previous iteration. Intuitively, as the upper-bound policy in Figure D.10 already prescribes the firm not to export to B at $t = 2$ at the path of interest, the change in the constant upper bound in A at $t = 2$ does not change the upper-bound policy function. Thus, at this point, the step 2 upper-bound policy function has converged; we represent it in Figure D.12.

Figure D.11: Updated Constant Upper Bounds That Condition on Path of Interest for $t \leqslant 2$

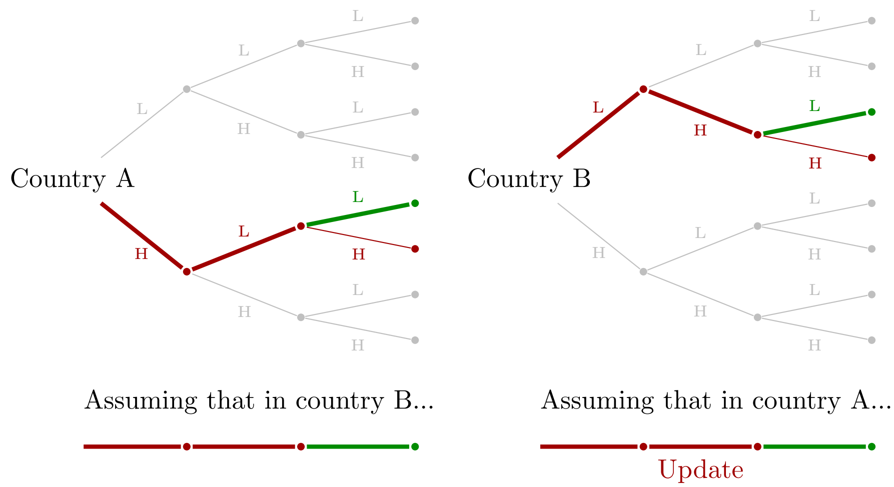
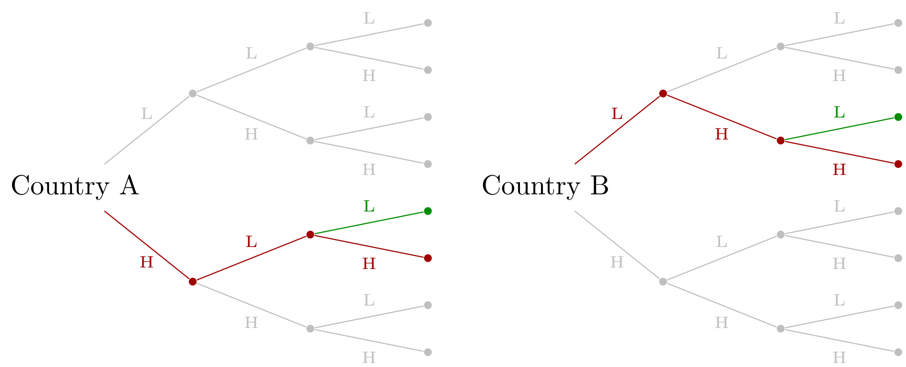


Figure D.12: Upper-Bound Policies That Condition on Path for $t \leqslant 2$ After Convergence



We follow similar steps to compute a lower-bound policy function that conditions on the state of interest at $t = 2$. As the lower-bound policy that converged in step 1 (see Figure D.7) prescribe the firm not to export to any country at any period or state, the resulting constant bounds are

$$(\underline{b}^{[0]}_{iA2|2}, \underline{b}^{[0]}_{iA3|2}) = (0, 0) \qquad \text{and} \qquad (\underline{b}^{[0]}_{iB2|2}, \underline{b}^{[0]}_{iB3|2}) = (0, 0). \tag{D.17}$$

Given these, the lower-bound policy cannot be updated further; we represent it in Figure D.13.

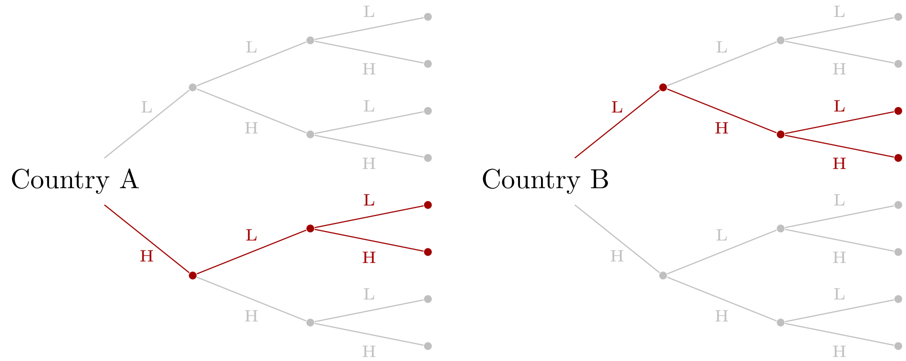
Evaluating the lower- and upper-bound policy functions in figures D.12 and D.13 at the path of interest at period $t = 2$, we obtain the following bounds on the firm's optimal export choices

$$\bar{\check{y}}_{iA2|2} = \underline{\check{y}}_{iA2|2} = 0 \qquad \text{and} \qquad \bar{\check{y}}_{iB2|2} = \underline{\check{y}}_{iB2|2} = 0. \tag{D.18}$$

As the bounds coincide, the firm's choice at $t = 2$ at the path of interest is $(\check{y}_{iA2}, \check{y}_{iB2}) = (0, 0)$.

Additional steps. At this point in the algorithm, we have computed the firm's optimal choice at the path of interest for $t \leqslant 2$. However, the step 1 bounds, described in equations (D.13) and (D.14) differ at the path of interest at $t = 3$. Our algorithm proceeds by trying to tighten these bounds. To do so, we first implement a step 2 procedure analogous to the one just described, but

Figure D.13: Lower-Bound Policies That Condition on Path at $t = 2$ After Convergence



now conditioning on the state of interest at $t = 3$. To save space, we do not describe here how the step 2 algorithm is applied at $t = 3$. It suffices to say that it is not successful at tightening further the bounds on the firm's optimal choice at the state of interest at $t = 3$. Thus, we proceed to the extra steps described in Appendix D.1. Specifically, computing the firm's choice at the state of interest at $t = 3$ requires solving jointly for the firm's optimal choices in A and B at this period.

D.3 Performance of the Algorithm

We present here summary statistics of the performance of the algorithm described in Section 4 and Appendix D.1. For all 4,709 firms in the sample, all 74 foreign countries we use in our estimation, 13 periods, and 5 simulation draws of ω_{ijt} for each i, j and t, we measure at the end of each step of the algorithm the percentage of all 22,650,290 ($4,709 \times 74 \times 13 \times 5$) choices solved and the cumulated running time (measured at Princeton University's Della cluster using 44 processors with 20 GB of memory per processor).

The statistics in Table D.1 are computed setting all parameter values to the baseline estimates reported in tables F.3 and F.4 in Appendix F.6. Step 1 of the algorithm (see Section 4) runs in slightly over two minutes, and provides the solution to 99.72% of the 22,650,290 choices considered. The 0.28% of choices that remain unsolved after step 1 are concentrated in a few countries but dispersed across firms and simulation draws; thus, the number of firms and draws whose choices in every country and period are solved in step 1 is only 78.51%.

Steps 2 and 3 increase the overall share of choices solved to 99.85%, and the share of firms and draws whose choices are completely solved to 93.07%. This is attained with a small cost in terms of computing time, as step 3 is completed after less than 4 minutes of running time. In steps 4 and 5, we solve optimization problems that consider multiple countries simultaneously. These steps are the slowest ones: approximately 70% of the 741 seconds it takes to run completely our algorithm are spent in steps 4 and 5. These steps are however useful at raising the share of choices solved to nearly 99.9%, and the share of firms and simulations entirely solved to nearly 96%.

The choices that remain unsolved after step 5 is finished are concentrated in countries that share cross-country complementarities with a large set of other potential export destinations. E.g., of all unsolved choices, nearly 7% are for Mexico, close to 6.5% are for Belgium, between 5% and 6% correspond to The Netherlands and Germany, and between 4% and 5% correspond to Sweden and France. These are all countries that share deep PTA (or regulatory proximity) with a number of other countries larger than the cardinality of the sets of destinations that we solve jointly in steps 4 and 5 of our algorithm.

Table D.1: Performance of Algorithm at Baseline Estimates

	Percentage of Firms Solved	Percentage of Choices Solved	Time (in seconds)
Step 1	78.51%	99.72%	131
Step 2	82.74%	99.75%	163
Step 3	93.07%	99.85%	218
Steps 4 & 5	95.80%	99.89%	741

In Table D.2, we present statistics analogous to those presented in the last row of Table D.1, but for alternative parameterizations in which we change the value of the model parameters one at a time. Specifically, we present results for parameterizations in which we increase in 20% the value of the parameter indicated in the column labeled "Parameter," leaving all other parameters at their baseline estimates.

The results in Table D.2 show the performance of the algorithm improves (i.e., the percentage of firms and simulations for which all choices are solved increases, and the running time decreases) as we increase the value of those parameters that have a positive impact on the gravity component of fixed and sunk costs; i.e., the parameters entering the expressions in equations (7) and (12). Conversely, the performance of the algorithm worsens as we increase the value of the parameters that have a positive impact on the magnitude of the complementarities between countries (i.e., $(\gamma^E_x, \varphi^E_x)$ for $x = \{g, l, a\}$), and improves as we increase the value of the parameters that determine the speed at which the complementarities between any two countries decay in the distance between them (i.e., κ^E_x for $x = \{g, l, a\}$). The performance of the algorithm varies very little with the value of the parameters that determine the cross-country correlation in the fixed cost shock ν_{ijt}; i.e., the parameters entering the expression in equation (10c). Finally, when we increase the standard deviation of ν_{ijt} or the probability that ω_{ijt} equals $\underline{\omega} = 0$ (i.e., when we increase σ_ν or p), the performance of the algorithm worsens.

Table D.2: Performance of Algorithm at Estimates 20% Higher than Baseline Ones

Parameter	Percentage of Firms Solved	Time (in seconds)	Parameter	Percentage of Firms Solved	Time (in seconds)
γ^F_0	97.18%	606	κ^E_l	96.03%	703
γ^F_g	97.25%	479	γ^E_a	91.28%	1256
γ^F_l	95.89%	710	φ^E_a	94.70%	935
γ^F_a	96.21%	628	κ^E_a	96.35%	647
γ^S_0	96.77%	582	γ^N_g	95.67%	795
γ^S_g	96.59%	569	κ^N_g	95.86%	742
γ^S_l	95.80%	719	γ^N_l	95.67%	687
γ^S_a	95.96%	692	κ^N_l	95.83%	689
γ^E_g	93.27%	1119	γ^N_a	95.77%	702
φ^E_g	93.59%	1070	κ^N_a	95.81%	686
κ^E_g	97.33%	479	σ_ν	93.88%	841
γ^E_l	95.52%	790	p	82.29%	2841
φ^E_l	95.65%	749			

Note: The *Percentage of Firms Solved* and *Time* are measured after step 5 of the algorithm has concluded.

E General Problem: Mapping to Model and Proofs

E.1 Mapping Between Framework in Appendix A.1 and Model

We show in this section that, equating agents to firms and alternatives to potential export destinations, the model described in Section 3 satisfies all restrictions in Assumption 1.

As part of the *first restriction*, equation (A.4) assumes agents maximize the expected infinite-horizon discounted sum of a sequence of static payoffs that exhibit one-period dependence. Equation (A.5) restricts these payoffs to be additively separable across alternatives and, in every alternative j, additively separable in the vector of shocks $\omega(z^t)$ and in the vector of choices in every alternative other than j. Finally, the restriction that the domain of the functions $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ is finite and that these never equal infinity in their domain implies both $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ are bounded from above. Additionally, $\tilde{\pi}_{jt}$ is also bounded from below.

Our model satisfies the first restriction in Assumption 1. Specifically, equation (A.4) is satisfied as equation (17) implies firms maximize the infinite-horizon expected discounted sum of static profits. Equation (A.5) is also satisfied as equations (13) to (15) imply that model-implied static profits are

$$\pi_t(y(z^t), y(z^{t-1}), \omega(z^t)) = \sum_{j=1}^{J} \{y_j(z^t) u_{jt}(y_j(z^{t-1}), \omega_j(z^t)) + \sum_{j'=1}^{J} y_j(z^t) y_{j'}(z^t) c_{jj't}\}, \tag{E.1}$$

where $\omega(z^t)$ equals a vector $(\omega_1(z^t), \ldots, \omega_J(z^t))$, $c_{jj't}$ is defined in equation (9) for $j' \neq j$ (with $c_{jjt} = 0$), and u_{jt} is defined in equation (14). Static profits may thus be written as in equation (A.5) with

$$\begin{aligned} \hat{\pi}_{jt}(y_j(z^t), y_j(z^{t-1}), \omega(z^t)) = {} & y_j(z^t)(\eta^{-1} \exp(\alpha_y y_j(z^{t-1}) + \alpha_{jt} + \alpha_s + \alpha_a \ln(a_{sjt}) + \alpha_r \ln(r_{iht})) \\ & - (g_{jt} + \nu_{ijt} + \omega_j(z^t)) - (1 - y_j(z^{t-1})) s_{jt}), \end{aligned} \tag{E.2a}$$

$$\tilde{\pi}_{jt}(y(z^t), y(z^{t-1})) = \sum_{j'=1}^{J} y_j(z^t) y_{j'}(z^t) c_{jj't}. \tag{E.2b}$$

Finally, these model-implied functions $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ satisfy the restrictions on their domain and range imposed in Assumption 1. Specifically, as $y_j(z^t) \in \{0, 1\}$, $y_j(z^{t-1}) \in \{0, 1\}$ and $\omega_j(z^t) \in \{0, \infty\}$ for all j and t, $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ are bounded from above for any realization of ν_{jt} as long as the parameter space is finite.[24]

The *second restriction* in Assumption 1 imposes the function π_t is supermodular on the sets of choices at $t-1$ and t. As these sets are finite, Corollary 2.6.1 in Topkis (1998) implies one can prove π_t is supermodular by proving it has increasing differences in $y(z^t)$ and $y(z^{t-1})$. For any alternative j and period t, we denote as D_{jt} the change in π_t when changing the value of the choice in j at t, y_{jt}, from zero to one. Given equations (E.1) and (E.2), the expression for D_{jt} in the model described in Section 3 is

$$\begin{aligned} D_{jt} = {} & \eta^{-1} \exp(\alpha_y y_j(z^{t-1}) + \alpha_{jt} + \alpha_s + \alpha_a \ln(a_{sjt}) + \alpha_r \ln(r_{ht})) \\ & - (g_{jt} + \nu_{jt} + \omega_j(z^t)) - (1 - y_j(z^{t-1})) s_{jt} + \sum_{j' \neq j} y_{j'}(z^t)(c_{jj't} + c_{j'jt}). \end{aligned}$$

[24] As equation (E.2a) shows, the model-implied function $\hat{\pi}_{jt}$ depends on $\omega(z^t)$ only through a scalar $\omega_j(z^t)$. While this is not relevant for the algorithm's theoretical properties (and, thus, is not imposed in Assumption 1), it is critical for its computational tractability.

Since $\alpha_y \geqslant 0$ and $s_{jt} \geqslant 0$ for every j and t, D_{jt} is increasing in $y_j(z^{t-1})$. Since $c_{jj't} \geqslant 0$ for any j, j', and t, D_{jt} is also increasing in $\{y_{j'}(z^t)\}_{j' \neq j}$. Finally, D_{jt} is invariant to $y_{j'}(z^{t-1})$ if $j' \neq j$. Thus, π_t has increasing differences on the sets of export choices at $t-1$ and t and, consequently, π_t is supermodular on these sets. The second restriction in Assumption 1 is thus satisfied by the model described in Section 3.

The *third restriction* in Assumption 1 imposes that there exists a feasible strategy such that, if chosen by the agent, the functions $\{\hat{\pi}_{jt}\}_j$ entering static profits are bounded from below no matter the value of the shock ω_t. In the model in Section 3, not exporting to country j ensures $\hat{\pi}_{jt}$ equals zero; i.e., $\hat{\pi}_{jt}(0, x, \omega) = 0$ for any $x \in \{0, 1\}$ and $\omega \in \Omega_t$. Thus, the third restriction in Assumption 1 is satisfied.

The *fourth restriction* imposes Ω_t is finite and the sequence of shocks $\{\omega_{jt}\}_{t \geqslant 0}$ is Markovian. In the model in Section 3, Ω_t includes only two elements and ω_t is independent over time (see equation (11)); thus, this fourth restriction is satisfied.

Finally, the *fifth restriction* imposes that the firm's problem becomes stationary after a terminal period T; i.e., the functions $\{\hat{\pi}_{jt}\}_j$ and $\{\tilde{\pi}_{jt}\}_j$, the distribution of ω_t, and the set Ω_t become constant at T. In the model described in Section 3, Ω_t and the distribution of ω_t are time-invariant, and the functions $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ become constant at T for every country j. Thus, the fifth restriction in Assumption 1 is satisfied.

E.2 Proof of Theorem 1: Preliminary Results

We prove here two preliminary results that we use in Appendix E.3 as part of the proof of Theorem 1. First, we show that restrictions 1 and 2 in Assumption 1 imply that the solution to the optimization problem in equation (A.6) for any given set of alternatives M_u is increasing in the second argument of the objective function Π_0; i.e., increasing in the upper bounds on the firm's optimal choice in every alternative not in M_u. Second, we show restrictions 1 and 3 to 5 in Assumption 1 imply there exists a solution to the optimization problem in equation (A.6), and that it attains the maximum. Additionally, we provide an algorithm to compute this solution. Finally, as a corollary, we show the solution of the optimization problem in equation (A.2) exists and the maximum is attained.

In our proofs, we use Lemma 2.6.1 and Theorem 2.8.1 in Topkis (1998), which we re-state here.

Lemma E.1 *(Topkis, 1998, Lemma 2.6.1) Suppose X is a lattice. Then,*

1. *If $f(x)$ is supermodular on X and $\alpha > 0$, then $\alpha f(x)$ is supermodular on X.*
2. *If $f(x)$ and $g(x)$ are supermodular on X, then $f(x) + g(x)$ is supermodular on X.*
3. *If $f_k(x)$ is supermodular on X for $k = 1, 2, \ldots$ and $\lim_{k\to\infty} f_k(x) = f(x)$ for each $x \in X$, then $f(x)$ is supermodular on X.*

Theorem E.1 *(Topkis, 1998, Theorem 2.8.1) If X is a lattice, T is a partially ordered set, S_t is a subset of X for each t in T, S_t is increasing in t on T, $f(x,t)$ is supermodular in x on X for each t in T, and $f(x,t)$ has increasing differences in (x,t) on $X \times T$, then $argmax_{x \in S_t} f(x,t)$ is increasing in t on $\{t : t \in T, argmax_{x \in S_t} f(x,t)$ is non-empty$\}$.*

E.2.1 First Preliminary Result

We prove here that, for any set of alternatives M_u and iteration n, if it exists, the solution $\bar{\boldsymbol{o}}^{(n)}_{M_u}$ to the optimization problem in equation (A.6) is increasing in the set of upper bounds on alternatives not in M_u; i.e., the solution to the optimization problem in equation (A.6) is increasing in

$$\bar{\boldsymbol{y}}^{(n)}_{M_u^c} = \{\bar{y}^{(n)}_{M_u^c}(z^t)\}_{t=0,z^t\in Z^t}^{\infty}, \qquad \text{with} \qquad \bar{y}^{(n)}_{M_u^c}(z^t) \geqslant o_{M_u^c}(z^t) \text{ for all } t \geqslant 0 \text{ and } z^t \in Z^t.$$

The proof has two steps. First, we show the agent's objective function according to equation (A.2), $\Pi_0(\boldsymbol{y})$, is supermodular in $\boldsymbol{y}$ on Y; see equation (A.1) for the definition of Y. Second, we show this implies that the solution to the optimization problem in equation (A.6) is increasing in the set of upper bounds on alternatives not in M_u.

Lemma E.2 *Assumption 1 implies $\Pi_0(\boldsymbol{y})$ is supermodular in $\boldsymbol{y}$ on Y.*

Proof. The second restriction in Assumption 1 in Appendix A.1 states that, for every period t and every feasible history z^t, $\pi_t(y(z^t), y(z^{t-1}), \omega(z^t))$ is supermodular in $(y(z^t), y(z^{t-1}))$ on $\{0,1\}^J \times \{0,1\}^J$. Define $\breve{\pi}_t(\boldsymbol{y}, z^t) = \pi_t(y(z^t), y(z^{t-1}), \omega(z^t))$, where, as indicated in Appendix A.1, $\boldsymbol{y}$ is a generic vector of agent choices at every history $z^t \in Z^t$ and every period $t \geqslant 0$. Therefore, $\breve{\pi}_t(\cdot)$ is identical to $\pi_t(\cdot)$, but written as a function of the whole vector of choices in every period and feasible history.

First, we show $\breve{\pi}_t(\boldsymbol{y}, z^t)$ is supermodular in $\boldsymbol{y}$. Specifically, we show that, for all $\boldsymbol{y}' \in Y$ and $\boldsymbol{y}'' \in Y$, it holds $\breve{\pi}_t(\boldsymbol{y}', z^t) + \breve{\pi}_t(\boldsymbol{y}'', z^t) \leqslant \breve{\pi}_t(\boldsymbol{y}' \vee \boldsymbol{y}'', z^t) + \breve{\pi}_t(\boldsymbol{y}' \wedge \boldsymbol{y}'', z^t)$, where the "join" $\vee$ takes the maximum element by element, and the "meet" $\wedge$ takes the minimum element by element. To prove this result, note that

$$\begin{aligned}
\breve{\pi}_t(\boldsymbol{y}', z^t) + \breve{\pi}_t(\boldsymbol{y}'', z^t) &= \pi_t(y'(z^t), y'(z^{t-1}), \omega(z^t)) + \pi_t(y''(z^t), y''(z^{t-1}), \omega(z^t)) \\
&\leqslant \pi_t(y'(z^t) \vee y''(z^t), y'(z^{t-1}) \vee y''(z^{t-1}), \omega(z^t)) \\
&\quad + \pi_t(y'(z^t) \wedge y''(z^t), y'(z^{t-1}) \wedge y''(z^{t-1}), \omega(z^t)) \\
&= \breve{\pi}_t(\boldsymbol{y}' \vee \boldsymbol{y}'', z^t) + \breve{\pi}_t(\boldsymbol{y}' \wedge \boldsymbol{y}'', z^t),
\end{aligned}$$

where the two equalities follow from the relationship between the functions π_t and $\breve{\pi}_t$, and the inequality follows from the supermodularity of $\pi_t(y'(z^t), y'(z^{t-1}), \omega(z^t))$ in $\{y(z^t), y(z^{t-1})\}$ on $\{0,1\}^J \times \{0,1\}^J$.

Second, we define a function $\Pi_0^\tau(\boldsymbol{y})$ as the expected discounted sum of static profits between periods $t = 0$ and $t = \tau$, and show that the supermodularity of $\breve{\pi}_t(\boldsymbol{y}, z^t)$ in $\boldsymbol{y}$ on Y implies $\Pi_0^\tau(\boldsymbol{y})$ is supermodular in $\boldsymbol{y}$ on Y. As the set Ω_t is finite for every period t (see restriction 4 in Assumption 1), we can write

$$\begin{aligned}
\Pi_0^\tau(\boldsymbol{y}) &= \pi_0(y(z^0), 0_J, \omega(z^0)) + \sum_{t=1}^{\tau} \sum_{z^t \in Z^t} \delta^t \pi_t(y(z^t), y(z^{t-1}), \omega(z^t)) \Pr(z^t), \\
&= \breve{\pi}_0(\boldsymbol{y}, z^0) + \sum_{t=1}^{\tau} \sum_{z^t \in Z^t} \delta^t \breve{\pi}_t(\boldsymbol{y}, z^t) \Pr(z^t).
\end{aligned}$$

Since $\breve{\pi}_t(\boldsymbol{y}, z^t)$ is supermodular in $\boldsymbol{y}$ on Y for every period t and history z^t, and the finite sum of supermodular functions is supermodular (see part 2 of Lemma E.1), then $\Pi_0^\tau(\boldsymbol{y})$ is supermodular in $\boldsymbol{y}$ on Y.

Finally, noting restriction 1 in Assumption 1 implies $\Pi_0(\boldsymbol{y}) = \lim_{\tau \to \infty} \Pi_0^\tau(\boldsymbol{y})$, we apply part 3 in Lemma E.1 to conclude that the supermodularity of $\Pi_0^\tau(\boldsymbol{y})$ in $\boldsymbol{y}$ on Y implies $\Pi_0(\boldsymbol{y})$ is supermodular in $\boldsymbol{y}$ on Y. ■

Lemma E.3 *Assumption 1 implies that, for every set of alternatives M_u and every iteration n of the algorithm described in Appendix A.2, if the solution to the optimization problem in equation* (A.6) *exists, it is increasing in the export strategy in every alternative not in M_u.*

Proof. This lemma states that, if it exists, $\bar{\boldsymbol{o}}^{(n)}_{M_u}$ is increasing in $\bar{\boldsymbol{y}}^{(n)}_{M_u}$. This lemma is implied by Theorem E.1 and the supermodularity of $\Pi(\boldsymbol{y})$ in $\boldsymbol{y}$ on Y. ■

E.2.2 Second Preliminary Result

We prove here that, for every subset of alternatives M_u and iteration n, the solution $\bar{\boldsymbol{o}}^{(n)}_{M_u}$ to the optimization problem in equation (A.6) exists and the maximum is attained. Specifically, Lemma E.4 below establishes the existence of the solution to the problem in equation (A.6), and that the maximum is attained, for every $t \geqslant T$; that, is, for all periods after the terminal period T, when the problem of the firm becomes stationary according to the restriction 5 in Assumption 1. Given Lemma E.4, establishing the existence of the solution to the problem in equation (A.6), and that the maximum is attained, for every $0 \leqslant t < T$ is straightforward by backward induction, as there are a finite number of feasible choices.

For any set of alternatives M_u and any vector $\bar{b}_{M_u^c} \in \{0,1\}^{J-J_u}$, we define the firm's expected discounted sum of static payoffs at T conditional on setting $\bar{y}_{M_u^c}(z^t) = \bar{b}_{M_u^c}$ for all $t \geqslant T$ and all $z^t \in Z^t$ as

$$\begin{aligned}\Pi_T\big(\boldsymbol{y}_{M_u}, \bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)\big) = {} & \pi_T\big((y_{M_u}(z^T), \bar{b}_{M_u^c}), (y_{M_u}(z^{T-1}), y_{M_u^c}(z^{T-1})), \omega(z^T)\big) \\ & + \sum_{t=T+1}^{\infty} \delta^{t-T}\mathbb{E}_T\Big[\pi_T\big((y_{M_u}(z^t), \bar{b}_{M_u^c}), (y_{M_u}(z^{t-1}), \bar{b}_{M_u^c}), \omega(z^t)\big)\Big],\end{aligned}$$

where $\pi_T(\cdot)$ equals the payoff function in equation (A.5) for $t = T$, $y(z^{T-1}) = (y_{M_u}(z^{T-1}), y_{M_u^c}(z^{T-1}))$, and $\boldsymbol{y}_{M_u}$ includes a generic set of choices for all alternatives in M_u, all $t \geqslant T$, and all $z^t \in Z^t$. We can then define the period-T value function

$$V_{TM_u}\big(\bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)\big) = \sup_{\boldsymbol{y}_{M_u}} \Pi_T\big(\boldsymbol{y}_{M_u}, \bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)\big). \tag{E.3}$$

Lemma E.4 *For any set of alternatives M_u and any vector $\bar{b}_{M_u^c} \in \{0,1\}^{J-J_u}$, Assumption 1 implies the solution to the problem in equation* (E.3) *exists and the maximum is attained.*

Proof. For any set of alternatives M_u and any vector $\bar{b}_{M_u^c} \in \{0,1\}^{J-J_u}$, we define the payoff function

$$\begin{aligned}&\check{\Pi}_T\big(\boldsymbol{y}_{M_u}, \bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)\big) = \\ &\quad \sum_{j \in M_u} \hat{\pi}_{jT}\big(y_j(z^T), y_j(z^{T-1}), \omega(z^T)\big) + \sum_{j=1}^{J} \tilde{\pi}_{jT}\big((y_{M_u}(z^T), \bar{b}_{M_u^c}), (y_{M_u}(z^{T-1}), y_{M_u^c}(z^{T-1}))\big) + \\ &\quad \sum_{t=T+1}^{\infty} \delta^{t-T}\mathbb{E}_T\Big[\sum_{j \in M_u} \hat{\pi}_{jT}\big(y_j(z^t), y_j(z^{t-1}), \omega(z^t)\big) + \sum_{j=1}^{J} \tilde{\pi}_{jT}\big((y_{M_u}(z^t), \bar{b}_{M_u^c}), (y_{M_u}(z^{t-1}), \bar{b}_{M_u^c})\big)\Big],\end{aligned}$$

and the associated value function

$$\check{V}_{TM_u}\big(\bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)\big) = \sup_{\boldsymbol{y}_{M_u}} \check{\Pi}_T\big(\boldsymbol{y}_{M_u}, \bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)\big). \tag{E.4}$$

The function $\check{\Pi}_T(\cdot)$ differs from $\Pi_T(\cdot)$ in that $\check{\Pi}_T(\cdot)$ only includes those terms entering $\Pi_T(\cdot)$ that depend on $\boldsymbol{y}_{M_u}$. Thus, $\check{\Pi}_T(\cdot)$ and $\Pi_T(\cdot)$ differ in a term that is invariant to the choice of $\boldsymbol{y}_{M_u}$ and, consequently, a vector $\boldsymbol{y}_{M_u}$ will solve the optimization problem in equation (E.4) if and only if it also solves the optimization problem in equation (E.3).

Restriction 1 in Assumption 1 implies the functions $\hat{\pi}_{jT}(\cdot)$ and $\tilde{\pi}_{jT}(\cdot)$ are bounded from above. As $\delta < 1$, we can then conclude that the value function $\check{V}_{TM_u}(\cdot)$ in equation (E.4) is bounded from above. Restriction 3 in Assumption 1 implies there is a feasible value of the choice vector $\boldsymbol{y}_{M_u}$ such that $\hat{\pi}_{jT}(\cdot)$ is bounded from below for all $j \in M_u$. As restriction 1 in Assumption 1 also implies that the function $\tilde{\pi}_{jT}(\cdot)$ is bounded from below, we can then conclude that the value function $\check{V}_{TM_u}(\cdot)$ in equation (E.4) is bounded from below. In sum, restrictions 1 and 3 in Assumption 1 imply that $\check{V}_{TM_u}(\cdot)$ is bounded from above and from below.

Theorem 4.2 in Stokey et al. (1989) implies we can write $\check{V}_{TM_u}(\cdot)$ as the solution to the following functional equation,

$$\check{V}_{TM_u}(\bar{b}_{M_u^c},(y_{M_u},y_{M_u^c}),\omega) =$$
$$\sup_{y'_{M_u}}\Big\{\sum_{j\in M_u}\hat{\pi}_{jT}(y'_j,y_j,\omega)+\sum_{j=1}^{J}\tilde{\pi}_{jT}\big((y'_{M_u},\bar{b}_{M_u^c}),(y_{M_u},y_{M_u^c})\big)\big)+\delta\mathbb{E}\big[\check{V}_{TM_u}(\bar{b}_{M_u^c},(y'_{M_u},\bar{b}_{M_u^c}),\omega)\big]\Big\} \tag{E.5}$$

Since $\check{V}_{TM_u}(\cdot)$ is bounded from above and from below, equation (E.5) maps bounded functions into bounded functions. Additionally, it also satisfies the monotonicity and discounting properties of Blackwell's sufficient conditions for a contraction of modulus δ. Therefore, there is a unique bounded function $\check{V}_{TM_u}(\cdot)$ that solves the problem in equation (E.5); see Theorem 3.3 in Stokey et al. (1989). Since the solution to the problem in equation (E.5) is unique, then it must also be a solution to the sequence problem in equation (E.4). Furthermore, as the solution to the sequence problems in equations (E.3) and (E.4) coincide, we can conclude that the solution to the optimization problem in equation (E.3) exists. Finally, as the choice variable y'_{M_u} in equation (E.5) may only take finitely many values, the maximum is attained. ■

Lemma E.5 *Assumption 1 implies the solution to the problem in equation* (A.2) *exists and the maximum is attained.*

Proof. It is an implication of Lemma E.4 when applied to the specific set M_u that includes all possible alternatives; i.e., $M_u = \{1,\ldots,J\}$. ■

E.3 Proof of Theorem 1

E.3.1 Proof of Part 1 of Theorem 1

We prove part 1 of Theorem 1 by induction.

As the *base case*, note that, according to equation (A.8), $\bar{b}_{jt}^{(1)} = 1$ for all $j = 1,\ldots,J$ and, therefore,

$$\bar{b}_{jt}^{(n)} \geqslant o_j(z^t) \qquad \text{for } n = 1,\ j = 1,\ldots,J,\ t \geqslant 0, \text{ and } z^t \in Z^t.$$

As the *step case*, suppose that, for some arbitrary n, $\bar{b}_{jt}^{(n)} \geqslant o_j(z^t)$ for all $j = 1,\ldots,J$, $t \geqslant 0$, and $z^t \in Z^t$. For any group of alternatives M_u, denote as

$$\bar{\boldsymbol{b}}^{(n)}_{M_u}$$

the vector that assigns the value of $\bar{b}^{(n)}_{jt}$ to every alternative j in M_u, every $t \geqslant 0$, and every $z^t \in Z^t$; i.e.,

$$\bar{\boldsymbol{b}}^{(n)}_{M_u} = \{\bar{y}^{(n)}_j(z^t)\}^{\infty}_{t=0,z^t\in Z^t,j\in M_u}, \quad \text{with} \quad \bar{y}^{(n)}_j(z^t) = \bar{b}^{(n)}_{jt} \text{ for all } t \geqslant 0, \text{ all } j \in M_u, \text{ and all } z^t \in Z^t.$$

Thus, $\bar{\boldsymbol{b}}^{(n)}_{M_u} \geqslant \boldsymbol{o}_{M_u}$, where $\boldsymbol{o}_{M_u}$ is the vector containing the agent's optimal choice for every $j \in M_u$, every $t \geqslant 0$, and every $z^t \in Z^t$. For any alternative j and period t, equations (A.6) and (A.9) further imply that

$$\bar{b}^{(n+1)}_{jt} = \max_{z^t\in Z^t} \bar{o}^{(n)}_j(z^t),$$

where, for a set M_u including alternative j, $\bar{o}^{(n)}_j(z^t)$ is the corresponding element of $\bar{\boldsymbol{o}}^{(n)}_{M_u}$, defined as

$$\bar{\boldsymbol{o}}^{(n)}_{M_u} = \underset{\boldsymbol{y}_{M_u}\in Y_{M_u}}{\operatorname{argmax}}\ \Pi_0(\boldsymbol{y}_{M_u}, \bar{\boldsymbol{b}}^{(n)}_{M^c_u}).$$

To prove that $\bar{b}^{(n+1)}_{jt} \geqslant o_j(z^t)$ for all $j = 1, \ldots, J$, $t \geqslant 0$, and $z^t \in Z^t$, it is thus enough to prove that

$$\bar{\boldsymbol{o}}^{(n)}_{M_u} \geqslant \boldsymbol{o}_{M_u}. \tag{E.6}$$

For any group of destinations M_u, we can write $\boldsymbol{o}_{M_u}$ as

$$\boldsymbol{o}_{M_u} = \underset{\boldsymbol{y}_{M_u}\in Y_{M_u}}{\operatorname{argmax}}\ \Pi_0(\boldsymbol{y}_{M_u}, \boldsymbol{o}_{M^c_u}). \tag{E.7}$$

Lemma E.4 implies $\bar{\boldsymbol{o}}^{(n)}_{M_u}$ and $\boldsymbol{o}_{M_u}$ exist, and Lemma E.3 implies $\bar{\boldsymbol{o}}^{(n)}_{M_u} \geqslant \boldsymbol{o}_{M_u}$. Thus, it holds that

$$\bar{b}^{(n+1)}_{jt} \geqslant o_j(z^t),$$

for all $j = 1, \ldots, J$, $t \geqslant 0$, and $z^t \in Z^t$. ■

E.3.2 Proof of Part 2 of Theorem 1

We prove part 2 of Theorem 1 by induction.

As *base case*, note that equation (A.8) implies $\bar{b}^{(1)}_{jt} = 1$ for every alternative j and period t. As, naturally,

$$\bar{o}^{(1)}_j(z^t) \in \{0, 1\}$$

for every alternative j, period $t \geqslant 0$, and history $z^t \in Z^t$, it must be the case that $\bar{b}^{(2)}_{jt}$, defined according to equation (A.9), is also either 0 or 1 for every alternative j and period t. Consequently,

$$\bar{b}^{(2)}_{jt} \leqslant \bar{b}^{(1)}_{jt}, \qquad \text{for all } j = 1, \ldots, J \text{ and } t \geqslant 0.$$

As the *step case*, suppose that, for some arbitrary n, $\bar{b}_{jt}^{(n)} \leqslant \bar{b}_{jt}^{(n-1)}$ for all $j = 1, \ldots, J$ and $t \geqslant 0$. Given the definition of $\bar{\boldsymbol{y}}_{M_u}^{(n)}$ in equation (A.7), it is then the case that, for any set of alternatives M_u, it holds that

$$\bar{\boldsymbol{y}}_{M_u}^{(n)} \leqslant \bar{\boldsymbol{y}}_{M_u}^{(n-1)}. \tag{E.8}$$

Given the definition of $\bar{\boldsymbol{o}}_{M_u}^{(n)}$ in equation (A.6), Lemma E.4 guarantees $\bar{\boldsymbol{o}}_{M_u}^{(n)}$ and $\bar{\boldsymbol{o}}_{M_u}^{(n-1)}$ exist. Given equations (A.6) and (E.8), Lemma E.3 implies that

$$\bar{\boldsymbol{o}}_{M_u}^{(n)} \leqslant \bar{\boldsymbol{o}}_{M_u}^{(n-1)}.$$

Since, according to equation (A.9), $\bar{b}_{jt}^{(n+1)} = \max_{z^t \in Z^t} \bar{o}_j^{(n)}(z^t)$ for every t, j, and z^t, it then holds that

$$\bar{b}_{jt}^{(n+1)} \leqslant \bar{b}_{jt}^{(n)},$$

for all $j = 1, \ldots, J$, $t \geqslant 0$, and $z^t \in Z^t$. ■

E.3.3 Proof of Part 3 of Theorem 1

As shown in the proof of Lemma E.4, Assumption 1 implies that, for any arbitrary iteration n, $\bar{b}_{jt}^{(n)} = \bar{b}_{jT}^{(n)}$ for every alternative j and period $t \geqslant T$; this is a consequence of the agent's optimization problem becoming stationary after period T. Therefore, we can summarize the infinite set of upper bounds

$$\{\bar{b}_{jt}^{(n)}\}_{j=1,t\geqslant T}^{J}$$

in a vector that belongs to the set $\{0,1\}^J$; i.e., in a vector with a finite number of coordinates. For every period $t < T$ and an arbitrary iteration n, it is the case that

$$\bar{b}_{jt}^{(n)} \in \{0,1\}^J.$$

Therefore, for any arbitrary iteration, computing the full set of upper-bounds $\{\bar{b}_{jt}^{(n)}\}_{j=1,t\geqslant 0}^{J}$ implies computing the value of $(T+1)J$ unknowns, each of whom may equal either 0 or 1.

Part 2 of Theorem 1 indicates that, at every iteration n, the value of each of these upper bounds either decreases or remains constant. As there is a finite number $(T+1)J$ of upper bounds to solve for at each iteration n, and each of these upper bounds may equal either 0 or 1 (i.e., they are bounded from below by 0), it must then be the case that these bounds converge in a finite number of steps.

F Estimation: Additional Details

F.1 Identification of Cross-Country Complementarities: Details

Consider a simplified version of the model described in Section 3 in which we impose the following restrictions. First, there are only three foreign countries. Second, in terms of the parameters entering the expression for potential export revenues in equation (5), assume that $\alpha_y = \alpha_a = \alpha_r = 0$, $\alpha_s = 0$ for every s, and, for every period t, $\alpha_{1t} = \alpha_{2t} = \bar{\alpha} = 1.05$ and $\alpha_{3t} = \alpha_3 = 1.15$. Third, in terms of the fixed export costs determined in equations (6) to (11), assume that

$$
\begin{aligned}
f_{i1t} &= \gamma_0^F + \nu_{i1t} + \omega_{i1t}, \\
f_{i2t} &= \gamma_0^F - y_{i3t}\bar{c} + \nu_{i2t} + \omega_{i2t}, \\
f_{i3t} &= \gamma_0^F - y_{i2t}\bar{c} + \nu_{i3t} + \omega_{i3t},
\end{aligned}
$$

with $\gamma_0^F = 80$, ν_{ijt} drawn according to the distribution in equation (10) with $\sigma_\nu = 80$ and, for all t,

$$
\rho_{12t} = \rho_{13t} = 0, \qquad \text{and} \qquad \rho_{23t} = \bar{\rho},
$$

and ω_{ijt} drawn according to the distribution in equation (11) with $p = 0.7$. Fourth, in terms of the sunk cost in equation (12), assume that, for every $j \in \{1, 2, 3\}$ and period t, $s_{jt} = \gamma_s^0 = 120$.

In this setting, we first show how the values of the moments $\mathbb{m}_1$ and $\mathbb{m}_2$ in equation (25) change as we change the value of the parameter determining the strength of the complementarities between countries 2 and 3 (i.e., $\bar{c}$), and the correlation coefficient in ν_{ijt} between countries 2 and 3 (i.e., $\bar{\rho}$). To do so, given values of $(\bar{c}, \bar{\rho})$, we simulate the model for 500 simulations of each of the 4,709 firms in our sample, set $T = 120$ and, to obtain results robust to initial conditions and the value of T, compute $\mathbb{m}_1$ and $\mathbb{m}_2$ using the information on y_{ijt} only for periods $50 \leqslant t \leqslant 64$.

In Table F.1, we compute $\mathbb{m}_1$ and $\mathbb{m}_2$ for four different values of $(\bar{c}, \bar{\rho})$. When setting $(\bar{c}, \bar{\rho}) = (0, 0)$, we obtain $\mathbb{m}_1 = \mathbb{m}_2 = 0$. As countries 1 and 2 are identical in every respect except in their potential complementarities with country 3, export probabilities in both countries must be equal when the parameter that determines the strength of those complementarities is set to 0; i.e., when $\bar{c} = 0$. Similarly, as all firms are identical in every respect except in the fixed cost unobserved terms ν_{ijt} and ω_{ijt}, the within-firm covariance in export choices in countries 2 and 3 will equal zero when the correlation in these unobserved terms for these two countries is zero; i.e., when $\bar{\rho} = 0$.

In the second row in Table F.1, we study a case with complementarities between countries 2 and 3; specifically, we set $\bar{c} = 30$. These complementarities increase the export probability in country 2 (and in country 3), while they do not affect the export probability in country 1 (as country 1 is isolated from any other potential export destination); therefore, $\mathbb{m}_1$ increases as $\bar{c}$ increases. As $\bar{c} > 0$, firms enjoy a reduction in fixed costs in country 2 if and only if they export in the same period to country 3 (and vice versa); thus, an increase in $\bar{c}$ makes firms more likely to simultaneously export to countries 2 and 3 and, consequently, $\mathbb{m}_2$ also increases as $\bar{c}$ increases.

In the third row in Table F.1, we study a case in which ν_{ijt} is positively correlated in countries 2 and 3; specifically, we set $\bar{\rho} = 0.8$. When there are no cross-country complementarities, the within-firm correlation in fixed costs in countries 2 and 3 does not affect the (marginal) export probability in any country; thus, $\mathbb{m}_1$ does not depend on the value of $\bar{\rho}$ when $\bar{c} = 0$. However, the within-firm positive correlation in fixed costs in countries 2 and 3 increases the probability that firms export simultaneously to those countries; thus, $\mathbb{m}_2$ increases in the value of $\bar{\rho}$ when $\bar{c} = 0$.

In the fourth row in Table F.1, we set $\bar{c}$ and $\bar{\rho}$ to positive values. When comparing the results in the second and fourth rows, we observe that introducing a positive correlation in ν_{ijt} between countries 2 and 3 in a setting with cross-country complementarities (i.e., when $\bar{c} > 0$) affects not only the probability firms export simultaneously to countries 2 and 3 (i.e., the value of $\mathbb{m}_2$) but also the difference in the export probabilities to countries 2 and 3 (i.e., the value of $\mathbb{m}_1$).

In unreported results, we observe that the patterns in Table F.1 hold when we change the values of $\bar{c}$ and $\bar{\rho}$ between different numbers, and when we set the size of country 3, α_3, to different values.

Table F.1: Impact of Complementarities and Correlation in Unobservables on Moments

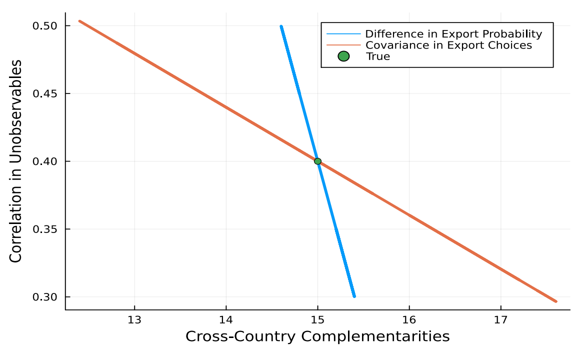
Parameters		Moments	
$\bar{c}$	$\bar{\rho}$	$\mathbb{E}[y_{i2t} - y_{i1t}]$	$\mathbb{C}[y_{i2t}, y_{i3t}]$
0	0	0	0
Positive	0	0.15	0.05
0	Positive	0	0.02
Positive	Positive	0.17	0.07

Note: by the label "Positive" in the first column, we denote cases in which $\bar{c} = 30$. By the label "Positive" in the second column, we denote cases in which $\bar{\rho} = 0.8$.

In Figure F.1, we perform a different exercise that more directly illustrates the capacity of $\mathbb{m}_1$ and $\mathbb{m}_2$ to identify the parameters $\bar{c}$ and $\bar{\rho}$. We simulate data from a "true" model in which we set $\alpha_3 = \bar{\alpha}$, $\bar{c} = 15$, and $\bar{\rho} = 0.4$, and we then compare how the values of moments $\mathbb{m}_1$ and $\mathbb{m}_2$ corresponding to the "true" model compare to those generated under alternative values of $\bar{c}$ and $\bar{\rho}$. More specifically, the green dot represents the true values of $\bar{c}$ and $\bar{\rho}$, and the blue and orange lines represent all values of $(\bar{c}, \bar{\rho})$ for which $\mathbb{m}_1$ and $\mathbb{m}_2$, respectively, equal their respective values in the "true" model. The slope of the orange line, e.g., shows we can keep moment $\mathbb{m}_2$ at its true value as we increase the value of the parameter $\bar{\rho}$ if we simultaneously decrease the value of the parameter $\bar{c}$. The blue line indicates the same is true for moment $\mathbb{m}_1$. Thus, neither moment alone allows to identify the parameter vector $(\bar{\rho}, \bar{c})$, but the fact that the orange and blue lines have different slopes implies that both moments jointly identify $(\bar{\rho}, \bar{c})$.

In unreported results, we observe the patterns in Figure F.1 hold when we change the values of $\bar{c}$ and $\bar{\rho}$ between different numbers, and when we set the size of country 3, α_3, to different values.

Figure F.1: Impact of Complementarities and Correlation in Unobservables on Moments



Notes: The axis labeled "Correlation in Unobservables" includes values of the parameter $\bar{\rho}$. The axis labeled "Cross-country Complementarities" includes values of the parameter $\bar{c}$. The green dot represents the true values of the parameters $\bar{c}$ and $\bar{\rho}$; i.e., $(\bar{c}, \bar{\rho}) = (15, 0.4)$. The blue and orange lines represent all values of $(\bar{c}, \bar{\rho})$ for which $\mathbb{m}_1$ and $\mathbb{m}_2$, respectively, equal their respective values in the "true" model.

F.2 Export Potential Measures

We define export potential in Appendix F.2.1. In Appendix F.2.2, we present summary statistics on the gravity equation estimates used to compute these export potentials, on the resulting export potential measures, and on the aggregate export potential of the countries geographically or linguistically close to each destination j, or that share a deep PTA with it. In Appendix F.2.3, we present reduced-form evidence showing firm export choices in a destination correlate with the aggregate export potential of the other countries that are geographically or linguistically close to it, or that share a deep PTA with it.

F.2.1 Definiton and Estimation of Export Potential Measures

We use country-to-country sector-specific trade flows, and the distance measures in Section 2, to compute measures of the *export potential* of Costa Rica in each sector, destination and year.[25] Specifically, we first compute Poisson pseudo-maximum-likelihood estimates of the parameters of the gravity equation

$$X^s_{odt} = \exp(\Psi^s_{ot} + \Xi^s_{dt} + \lambda^s_g n^g_{od} + \lambda^s_l n^l_{od} + \lambda^s_a n^a_{odt}) + u^s_{odt}, \tag{F.1}$$

where X^s_{odt} denotes the export volume from origin o to destination d in sector s and year t; Ψ^s_{ot} and Ξ^s_{dt} are sector-origin-year and sector-destination-year unobserved effects, respectively; n^g_{od}, n^l_{od}, and n^a_{odt} are the distance measures described in Section 2; λ^s_g, λ^s_l, and λ^s_l are sector-specific parameters; and u^s_{odt} is an unobserved term. Denoting parameter estimates with a hat, we measure Costa Rica's export potential in a sector s, destination j, and year t as

$$E^s_{jt} = \exp(\hat{\Xi}^s_{jt} + \hat{\lambda}^s_g n^g_{hj} + \hat{\lambda}^s_l n^l_{hj} + \hat{\lambda}^s_a n^a_{hjt}), \tag{F.2}$$

where n^g_{hj}, n^l_{hj}, and n^a_{hjt} denote distances between Costa Rica and country j.[26]

F.2.2 Gravity-Equation Estimates and Export Potential Measures: Statistics

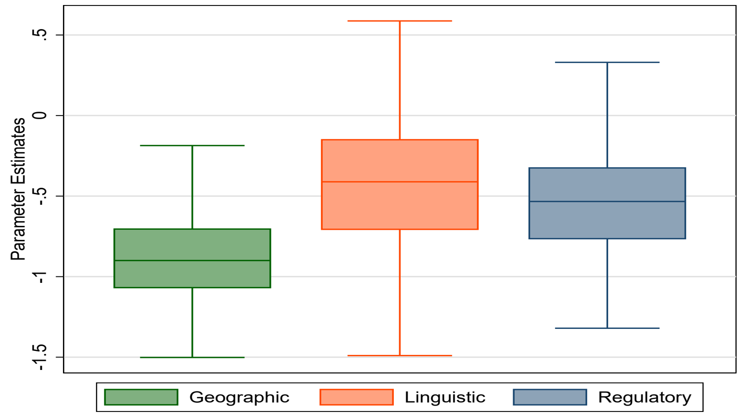
In Figure F.2, we include boxplots summarizing the distribution across sectors of the parameter estimates $\hat{\lambda}^s_g$ (in green), $\hat{\lambda}^s_l$ (in orange), and $\hat{\lambda}^s_a$ (in blue). The estimates of λ^s_g are negative for all sectors and centered around -1. The estimates of λ^s_l and λ^s_a are also nearly always negative, although they tend to be smaller in absolute value than the estimates of λ^s_g.

In Figure F.3, we present boxplots summarizing the distribution across sectors and years of the export potential measures E^s_{jt} for the ten destination countries with the largest (in Figure F.3a) and smallest (in Figure F.3b) mean export potentials. The US is the country with the largest mean value of E^s_{jt}. The distribution of E^s_{jt} for the US is actually distinctively different from that corresponding to all other destinations, with the first quartile of the distribution for the US being similar to the third quartile of the distribution of export potentials in Mexico, which is the country with the second largest mean export potential. Other destinations with large mean export potentials are

[25] The BACI data by CEPII reports country-to-country trade flows at the HS-6 level; see Gaulier and Zignago (2010) for details. Using a concordance provided by WITS (`https://wits.worldbank.org/product_concordance.html`), we aggregate this product-level data to generate sector-level flows, with sectors defined at the four-digit level according to ISIC Rev. 3. We use a concordance provided by UNSD (`https://unstats.un.org/unsd/classifications/Econ/ISIC.cshtml`) to further convert the data to four-digit sectors defined according to the ISIC Rev. 4.

[26] When estimating equation (F.1), we exclude observations in which Costa Rica is the origin or destination.

Figure F.2: Estimates of Gravity Equation Parameters



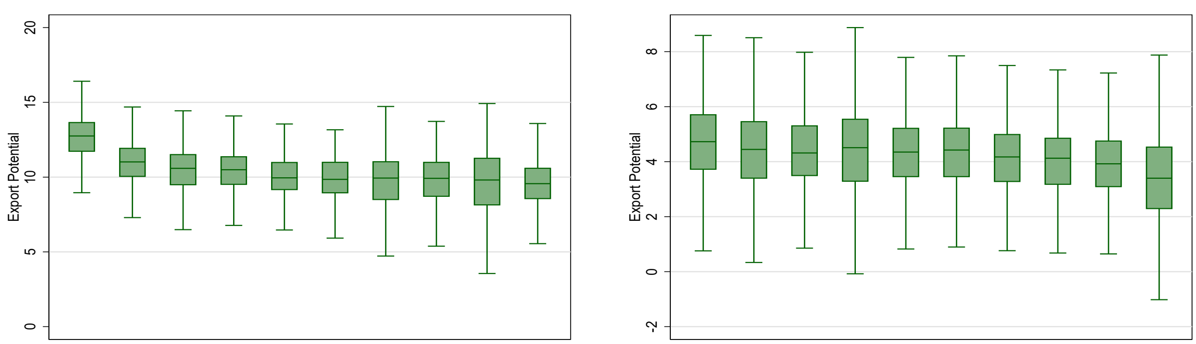
Notes: These boxplots represent the distribution of $\hat{\lambda}_g^s$ (geographic), $\hat{\lambda}_l^s$ (linguistic) and $\hat{\lambda}_a^s$ (regulatory) across sectors.

countries that are geographically or linguistically close to Costa Rica (e.g., Panama, Colombia, Venezuela, Spain), or countries that are large importers (e.g., Canada, Germany, Brazil, China). As Figure F.3b shows, the ten destination countries with the smallest mean export potentials (e.g., Bhutan, the Central African Republic, Seychelles, or Burundi) are all small, distant from Costa Rica geographically and linguistically, and do not share any PTA with Costa Rica.

Figure F.3: Export Potential - Distributions by Country for Top 10 Destinations

(a) Top 10 Destinations

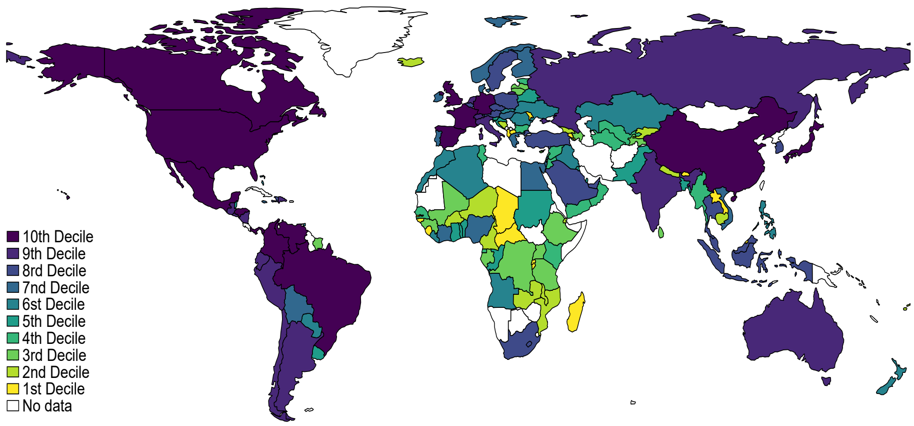
(b) Bottom 10 Destinations



Notes: These boxplots summarize the distribution of E_{jt}^s (see equation (F.2)) for the 10 destination countries with the largest (Figure F.3a) and smallest (Figure F.3b) mean export potentials, where the mean is computed across sectors and years in the period 2005-2015. Countries are listed according to their alpha-3 ISO code.

In Figure F.4, we show a map displaying, for each country j, the mean value of E_{jt}^s across the sectors and years in the sample. Most countries in North, Central, and South America, and in Europe, are in the top three deciles. Also in the top three deciles are Australia, Russia, China and India. On the contrary, most countries in Africa, several in South Asia, and the former Soviet republics are in the bottom deciles.

Figure F.4: Mean Export Potential by Destination Country



Notes: Map of the mean (across sectors and years in the period 2005-2015) E^s_{jt} by country.

For each foreign country j, sector s, and year t, we use the export potential measures E^s_{jt} of countries other than j to construct the aggregate export potential of the countries geographically or linguistically close to j, or that share a deep PTA with it. Denoting the aggregate export potential of the countries that, e.g., are geographically close to a destination j as $AE^s_{jt,g}$, we compute it as the sum of the sector- and year-specific export potentials of all countries whose geographical distance to j is smaller than some threshold $\bar{n}_g$:

$$AE^s_{jt,g} = \sum_{j' \neq j} \mathbb{1}\{n^g_{jj'} \leqslant \bar{n}_g\} E^s_{j't}. \tag{F.3}$$

We build similar measures for countries linguistically close to j, or cosignatories of a deep PTA with j, denoted respectively as $AE^s_{jt,l}$ and $AE^s_{jt,a}$. We use as thresholds $\bar{n}_g = 0.79$, $\bar{n}_l = 0.11$, or $\bar{n}_a = 0.43$.

We describe in Figure F.5 the mean (across sectors and years in the period 2005-2015) value of $AE^s_{jt,g}$ (panel (a)), $AE^s_{jt,l}$ (panel (b)), and $AE^s_{jt,a}$ (panel (c)), for every destination in the sample.

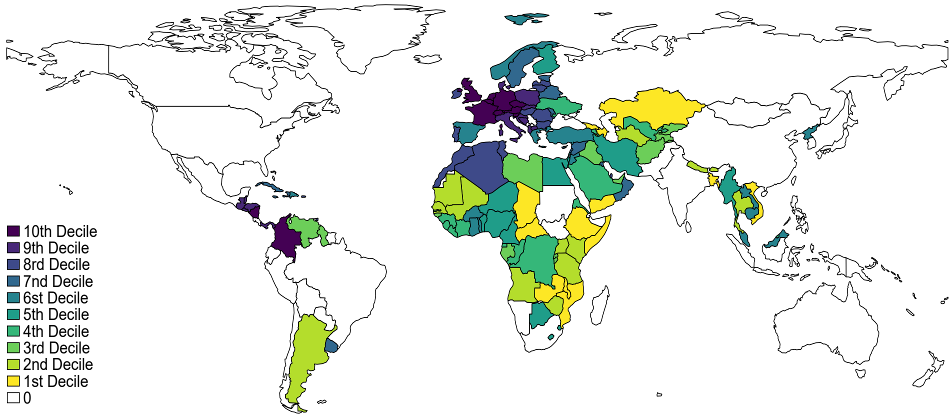
Our measure of the geographical distance between any two countries j and j' is a weighted average of the distances between cities located in j and j'. Large countries thus tend to be geographically isolated. This explains why the US, Canada, Russia, or China have a zero value of $AE^s_{jt,g}$; these countries have no other country with whom their bilateral geographic distance is below the threshold of 790 km we use in this figure to classify two countries as neighbors. Conversely, as illustrated in Figure F.5a, countries located in Central America and in Central Europe have many neighbors with relatively large export potentials. Thus, their value of $AE^s_{jt,g}$ is large. The aggregate export potential of neighbors is smaller for countries in Africa (which tend to have many neighbors, but small in terms of their own export potential).

The map in Figure F.5b shows that countries with a large share of Spanish speakers (Spain and several countries in South and Central America) and countries with a large share of English speakers (countries such as Australia and the UK, in which English is an official language, but also countries in which English is not an official language such as Germany or Denmark) exhibit large values of $AE^s_{jt,l}$.

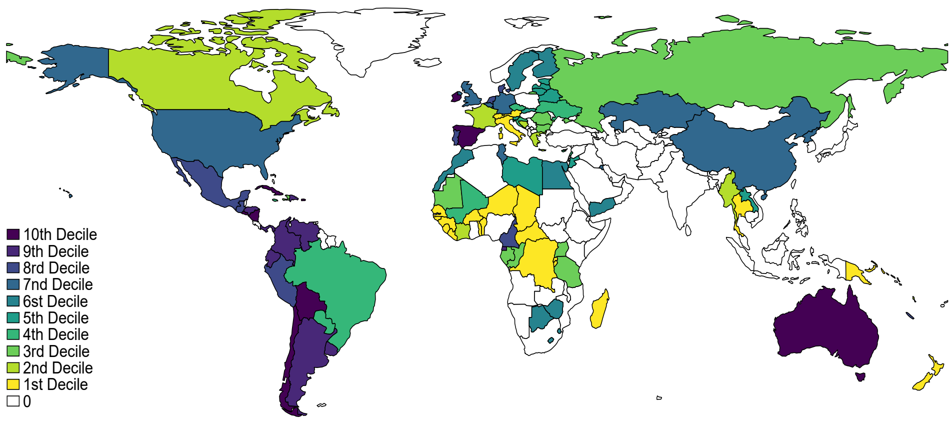
Finally, Figure F.5c shows that countries in the EU, NAFTA or CAFTA, and countries that have deep PTAs with these blocs (e.g., Morocco and Australia) have large values of $AE^s_{jt,a}$.

Figure F.5: Aggregate Export Potential Measures

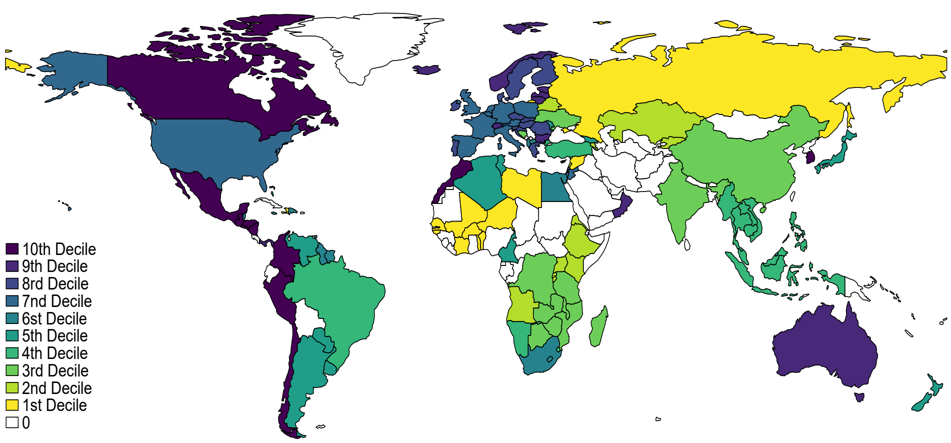
(a) Based on Geographic Distance



(b) Based on Linguistic Distance



(c) Based on Common Membership in a Deep PTA



Notes: Each panel displays the mean (across sectors and sample years) for each foreign country of the aggregate export potential measures $AE^s_{jt,g}$ (panel (a)), $AE^s_{jt,l}$ (panel (b)), and $AE^s_{jt,a}$ (panel (c)).

F.2.3 Correlation Between Export Potential Measures and Export Choices

As illustrated in Section 5.2.1, if geographical or linguistic proximity, or common participation in a deep PTA, are a source of cross-country complementarities in export participation decisions, a firm's export probability in a country j and year t will, all else equal, increase in the aggregate market size of the countries geographically or linguistically close to j, or that share a deep PTA with it. To test this implication, we use the aggregate export potential measures introduced above as proxy for the aggregate market size of the countries close to j, and compute OLS estimates of a regression of a dummy variable that equals one if firm i exports to country j in year t on flexible functions of country j's log export potential (introduced only as a control variable) and the log of the aggregate export potential of the countries geographically or linguistically close to j, or that share a deep PTA with it. Specifically, given the estimating equation

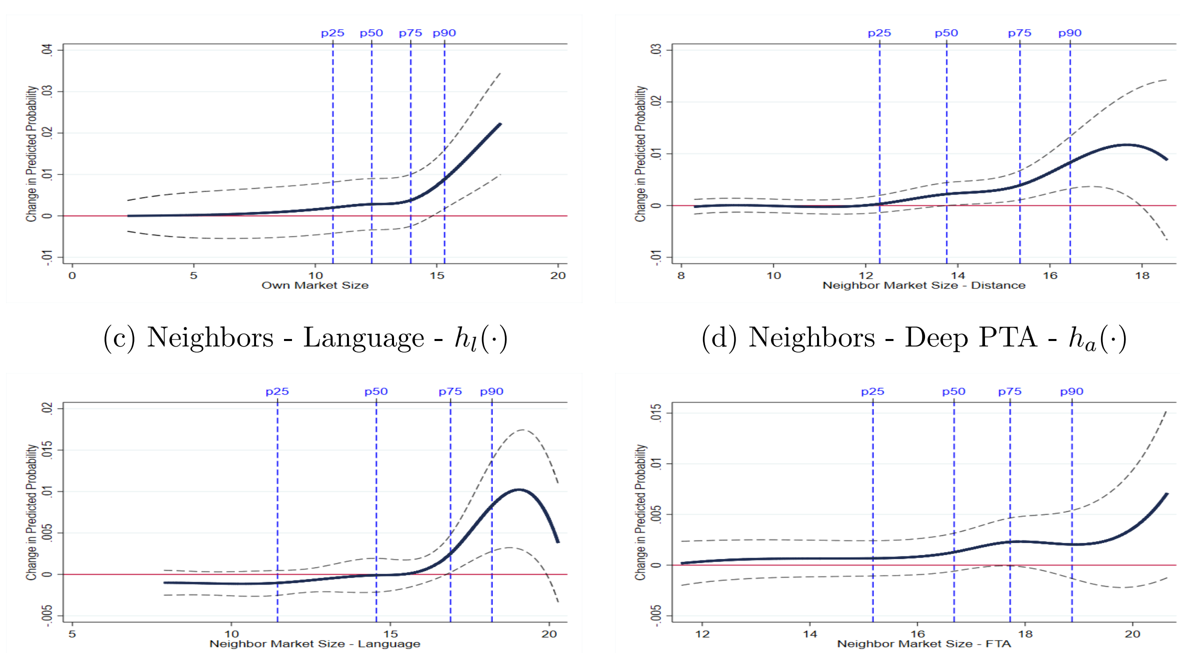
$$y_{ijt} = h_o(\ln(E_{sjt})) + \sum_{x=\{g,l,a\}} \mathbb{1}\{AE^s_{jt,x} > 0\} h_x(\ln(AE^s_{jt,x})) + \beta_{it} + u_{ijt}, \tag{F.4}$$

where $h_x(\cdot)$ for $x = \{o, g, l, a\}$ are cubic splines, and β_{it} is a firm-year fixed effect, panels (a) to (d) in Figure F.6 respectively show OLS estimates of the functions $h_o(\cdot)$, $h_g(\cdot)$, $h_l(\cdot)$, and $h_a(\cdot)$.

Figure F.6: Impact of Own and Neighbors' Export Potential

(a) Own - $h_o(\cdot)$ (b) Neighbors - Geography - $h_g(\cdot)$

(c) Neighbors - Language - $h_l(\cdot)$ (d) Neighbors - Deep PTA - $h_a(\cdot)$



Notes: Panels (a), (b), (c), and (d) show the point estimate and 95% confidence intervals for the cubic splines $h_o(\cdot)$, $h_g(\cdot)$, $h_l(\cdot)$, and $h_a(\cdot)$, respectively. The marks *p25*, *p50*, *p75*, and *p90* correspond to the 25th, 50th, 75th, and 90th percentiles of the corresponding covariate; i.e., E_{sjt} for panel (a), $AE^s_{jt,g}$ for panel (b), $AE^s_{jt,l}$ for panel (c), and $AE^s_{jt,a}$ for panel (d). Standard errors are clustered by country.

The estimates in Figure F.6 imply that the effect of a country's own export potential as well as the effect of the aggregate export potential of a country's neighbors is highly non-linear, with effects being generally not statistically different from zero until we reach the destination that is at the 75th percentile of the distribution of the corresponding variable. From the 75th percentile onwards, the firm's export probability in a destination increases in the destination's own export potential and in the aggregate export potential of the countries close to it.

F.3 List of Moment Conditions

As discussed in Section 5.2, our SMM estimator uses moments that take the form

$$\frac{1}{M}\sum_{i=1}^{M}\left\{m_k(y_i^{obs},z_i,x)-\frac{1}{S}\sum_{i=1}^{S}m_k(y_i^s(\theta),z_i,x)\right\}=0, \tag{F.5}$$

where y_i^{obs} includes the observed firm i's export participation decisions in every country j and in every sample period t in which the firm is active; z_i includes all observed payoff-relevant variables and all estimates computed in the first step of our estimation procedure (see Section 5.1); x includes the export potential measures in equation (F.2) for all foreign countries and sample periods; and $y_i^s(\theta)$ includes all model-implied export participation decisions for given values of z_i and the parameter vector θ, and a draw χ_i^s from the distribution of χ_i conditional on z_i. Specifically, we can write z_i, x, and χ_i as

$$\begin{aligned}
z_i \equiv\ &(\hat{\alpha}_y,\hat{\alpha}_a,\hat{\alpha}_r,\hat{\beta}_\alpha,\hat{\rho}_\alpha,\hat{\sigma}_\alpha,\hat{\beta}_r,\hat{\rho}_r,\hat{\sigma}_r,\{\hat{\alpha}_{jt}\}_{j=1,t=\underline{t}}^{J,\bar{t}},\hat{\alpha}_s,\{r_{iht}\}_{t=t_i}^{\bar{t}},\{a_{st}\}_{t=\underline{t}}^{\bar{t}},\{(n_{jj'}^g,n_{jj'}^l)\}_{j=1,j'=1}^{J,J},\\
&\{n_{jj't}^a\}_{j=1,j'=1,t=\underline{t}}^{J,J,\bar{t}},\{(n_{hj}^g,n_{hj}^l)\}_{j=1}^{J},\{n_{hjt}^a\}_{j=1,t=\underline{t}}^{J,\bar{t}}),\\
x =\ &\{E_{jt}^s\}_{j=1,t=\underline{t}}^{J,\bar{t}},\\
\chi_i \equiv\ &(\{\alpha_{jt}\}_{j=1,t=\underline{t}_i}^{J,\underline{t}-1},\{\alpha_{jt}\}_{j=1,t=\bar{t}+1}^{J,T},\{r_{iht}\}_{t=\underline{t}_i}^{\underline{t}-1},\{r_{iht}\}_{t=\bar{t}+1}^{T},\{\nu_{ijt}\}_{j=1,t=\underline{t}_i}^{J,T},\{\omega_{ijt}\}_{j=1,t=\underline{t}_i}^{J,\bar{t}}),
\end{aligned} \tag{F.6}$$

where s is i's sector, $\underline{t}$ and $\bar{t}$ are the first and last sample years, $\underline{t}_i$ is i's birth year, and $t_i=\max\{\underline{t},\underline{t}_i\}$.

Each moment function $m_k(\cdot)$ is an average over foreign countries and periods of a function $\tilde{m}_{k,jt}(\cdot)$. Specifically, both for $y_i=y_i^{obs}$ and for $y_i=y_i^s(\theta)$, it holds that

$$m_k(y_i,z_i,x)\equiv\frac{1}{J(\bar{t}-t_i)}\sum_{j=1}^{J}\sum_{t=t_i}^{\bar{t}}\tilde{m}_{k,jt}(y_i,z_i,x). \tag{F.7}$$

We use 89 moments as defined by equations (F.5) and (F.7). We classify them in three blocks.

The first block includes moments targeted to identify the parameters determining the level of fixed and sunk costs as well as the impact on them of the distance between the firm's home country and each destination. Specifically, the first block of moments targets the identification of the parameters

$$(\gamma_0^F,\gamma_0^S,\{(\gamma_x^F,\gamma_x^S)\}_{x=\{g,l,a\}}),$$

which enter the model through the expressions in equations (7) and (12). A first set of moments in this block captures firms' export participation choices by groups of destinations that differ in their distances to the firm's home country. More specifically, these moments are defined by the functions

$$\tilde{m}_{k,jt}(y,z,x)=y_{ijt}\mathbb{1}\{n_{hjt}^{x_1}<\bar{n}_{x_1}\}\mathbb{1}\{n_{hjt}^{x_2}<\bar{n}_{x_2}\}n_{hjt}^{x_1}n_{hjt}^{x_2}, \tag{F.8a}$$

$$\tilde{m}_{k,jt}(y,z,x)=y_{ijt}\mathbb{1}\{n_{hjt}^{x_1}\geqslant\bar{n}_{x_1}\}\mathbb{1}\{n_{hjt}^{x_2}<\bar{n}_{x_2}\}n_{hjt}^{x_1}n_{hjt}^{x_2}, \tag{F.8b}$$

$$\tilde{m}_{k,jt}(y,z,x)=y_{ijt}\mathbb{1}\{n_{hjt}^{x_1}<\bar{n}_{x_1}\}\mathbb{1}\{n_{hjt}^{x_2}\geqslant\bar{n}_{x_2}\}n_{hjt}^{x_1}n_{hjt}^{x_2}, \tag{F.8c}$$

$$\tilde{m}_{k,jt}(y,z,x) = y_{ijt}\mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} \geqslant \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.8d}$$

for (x_1, x_2) in $\{(g,l),(g,a),(l,a)\}$. As a reminder, n^g_{hj}, n^l_{hj}, and n^a_{hjt} respectively denote the geographic, linguistic and regulatory distances between the firm's home country h and the foreign country j. The constants $\bar{n}_{x_1}$ and $\bar{n}_{x_2}$ are thresholds that split foreign countries into two groups depending on whether their distance to the firm's home market h is larger or smaller than the corresponding threshold; specifically, we set $\bar{n}_g = 6$ (i.e., 6,000 km), $\bar{n}_l = 0.5$, and $\bar{n}_a = 1$. According to these thresholds, we split countries roughly depending on whether they are in the Americas (in which case $n^g_{hj} < 6$), on whether at least 50% of their population speak Spanish, and on whether they have any sort of deep PTA with Costa Rica (in which case $n^a_{hjt} < 1$). E.g., the moment defined by the function in equation (F.8a) for $(x_1, x_2) = (g, l)$ is

$$\frac{1}{M}\sum_{i=1}^{M}\Big\{\frac{1}{J(\bar{t}-t_i)}\sum_{j=1}^{J}\sum_{t=t_i}^{\bar{t}}(y^{obs}_{ijt} - \frac{1}{S}\sum_{i=1}^{S}y^s_{ijt}(\theta))\mathbb{1}\{n^g_{hj} < 6\}\mathbb{1}\{n^l_{hj} < 0.5\}n^g_{hj}n^l_{hj}\Big\} = 0. \tag{F.9}$$

For the foreign countries less than 6,000 km away from Costa Rica and with linguistic distance to Costa Rica below 0.5, this moment captures the average (across firms, countries and periods) difference between the observed firm choices and the average (across S simulated samples) choices implied by our model. When computing this average, each observation is weighted by the product of the geographic and linguistic distances of each destination to the firm's home country.

A second set of moments still within this first block are defined by the following functions:

$$\tilde{m}_{k,jt}(y_i,z_i,x) = y_{ijt}y_{ijt-1}\mathbb{1}\{n^{x_1}_{hjt} < \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} < \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.10a}$$
$$\tilde{m}_{k,jt}(y_i,z_i,x) = y_{ijt}y_{ijt-1}\mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} < \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.10b}$$
$$\tilde{m}_{k,jt}(y_i,z_i,x) = y_{ijt}y_{ijt-1}\mathbb{1}\{n^{x_1}_{hjt} < \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} \geqslant \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.10c}$$
$$\tilde{m}_{k,jt}(y_i,z_i,x) = y_{ijt}y_{ijt-1}\mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} \geqslant \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.10d}$$

for (x_1, x_2) in $\{(g,l),(g,a),(l,a)\}$. These functions differ from those in equation (F.8) in that they do not depend on whether firms export to a country j at a period t (as captured by y_{ijt}) but on whether they continue exporting at t to a country j to which it was exporting at $t-1$ (as captured by $y_{ijt}y_{ijt-1}$). E.g., the moment defined to the function in equation (F.10a) for $(x_1, x_2) = (g, l)$ is

$$\frac{1}{M}\sum_{i=1}^{M}\Big\{\frac{1}{J(\bar{t}-t_i)}\sum_{j=1}^{J}\sum_{t=t_i}^{\bar{t}}(y^{obs}_{ijt}y^{obs}_{ijt-1} - \frac{1}{S}\sum_{i=1}^{S}y^s_{ijt}(\theta)y^s_{ijt-1}(\theta))\mathbb{1}\{n^g_{hj} < 6\}\mathbb{1}\{n^l_{hj} < 0.5\}n^g_{hj}n^l_{hj}\Big\} = 0.$$

The interpretation of this moment is analogous to that in equation (F.9), with the only difference that it focuses in export survival events instead of export participation events.

Equations (F.8) and (F.10) list four moments each for each (x_1, x_2) in $\{(g,l),(g,a),(l,a)\}$. Thus, the first block of moments includes 24 moments in total.

The second block includes moments targeted to identify the parameters determining the strength of the complementarities. That is, this block of moments targets the identification of the parameters

$$\{(\gamma^E_x, \psi^E_x, \kappa^E_x)\}_{x=\{g,l,a\}},$$

which enter the model through the expression in equation (9). The functions defining the moments included in this second block capture firms' export probabilities by groups of destinations that differ in the aggregate export potential of the countries that are at a given geographical, linguistic, or regulatory distance to them. A key variable in these moments is thus the aggregate export potential of the countries that are within certain distance thresholds of each potential destination:

$$AE^{s,x_2}_{jt,x_1} = \sum_{j' \neq j} \mathbb{1}\{\underline{n}^{x_2}_{x_1} \leqslant n^{x_1}_{jj't} < \bar{n}^{x_2}_{x_1}\} E^s_{j't}, \tag{F.11}$$

where the index x_1 identifies the distance measure, and the index x_2 identifies the distance interval over which we are summing the export potential measures $E^s_{j't}$. The index x_1 takes values in the set $\{g, l, a\}$, with, e.g., $x_1 = g$ denoting the geographical distance in equation (B.1). The index x_2 takes values in the set $\{1, 2, 3\}$, and it determines the distance thresholds according to the following rules. For the case in which $x_1 = g$, the distance thresholds are

$$(\underline{n}^{x_2}_g, \bar{n}^{x_2}_g) = \begin{cases} (0, 426) & \text{if } x_2 = 1, \\ (426, 790) & \text{if } x_2 = 2, \\ (790, 1153) & \text{if } x_2 = 3. \end{cases} \tag{F.12}$$

For the case in which $x_1 = l$, the distance thresholds are

$$(\underline{n}^{x_2}_l, \bar{n}^{x_2}_l) = \begin{cases} (0, 0.01) & \text{if } x_2 = 1, \\ (0.01, 0.11) & \text{if } x_2 = 2, \\ (0.11, 0.50) & \text{if } x_2 = 3. \end{cases} \tag{F.13}$$

Finally, for the case in which $x_1 = a$, the distance thresholds are

$$(\underline{n}^{x_2}_a, \bar{n}^{x_2}_a) = \begin{cases} (0, \frac{1}{7}) & \text{if } x_2 = 1, \\ (\frac{1}{7}, \frac{3}{7}) & \text{if } x_2 = 2, \\ (\frac{3}{7}, \frac{6}{7}) & \text{if } x_2 = 3. \end{cases} \tag{F.14}$$

Then, e.g., the variables $AE^{s,1}_{jt,g}$, $AE^{s,2}_{jt,g}$, and $AE^{s,3}_{jt,g}$ denote the aggregate export potential in sector s and year t of all countries j' other than j that are less than 426 km away from j, between 426 km and 790 km away from j, and between 790 km and 1153 km away from j, respectively. Given AE^{s,x_2}_{jt,x_1} for $x_1 = \{g, l, a\}$ and $x_2 = \{1, 2, 3\}$, the moments in this second block are defined by

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \mathbb{1}\{n^{x_1}_{hjt} < \bar{n}_{x_1}\} \mathbb{1}\{AE^{s,x_2}_{jt,x_1} = 0\}, \tag{F.15a}$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \mathbb{1}\{n^{x_1}_{hjt} < \bar{n}_{x_1}\} \mathbb{1}\{0 < AE^{s,x_2}_{jt,x_1} \leqslant p_{66}(AE^{s,x_2}_{jt,x_1})\}, \tag{F.15b}$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \mathbb{1}\{n^{x_1}_{hjt} < \bar{n}_{x_1}\} \mathbb{1}\{p_{66}(AE^{s,x_2}_{jt,x_1}) < AE^{s,x_2}_{jt,x_1}\}, \tag{F.15c}$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\} \mathbb{1}\{AE^{s,x_2}_{jt,x_1} = 0\}, \tag{F.15d}$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\} \mathbb{1}\{0 < AE^{s,x_2}_{jt,x_1} \leqslant p_{66}(AE^{s,x_2}_{jt,x_1})\}, \tag{F.15e}$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\} \mathbb{1}\{p_{66}(AE^{s,x_2}_{jt,x_1}) < AE^{s,x_2}_{jt,x_1}\}, \tag{F.15f}$$

where $p_{66}(\cdot)$ is the 66th percentile of the random variable in parenthesis. For any $x_1 = \{g, l, a\}$, $n^{x_1}_{hjt}$ denotes the corresponding distance between the firm's home country h and destination j, and $\bar{n}_{x_1}$ is a threshold we use to split destinations into two groups depending on whether their distance to the home market is larger or smaller than the corresponding threshold; we set $\bar{n}_g = 6$, $\bar{n}_l = 0.5$, and $\bar{n}_a = 1$, which are the same thresholds we use to define the moments in equations (F.8) and (F.10). E.g., the moment given by the function in equation (F.15a) for $(x_1, x_2) = (g, 1)$ is

$$\frac{1}{M}\sum_{i=1}^{M}\Big\{\frac{1}{J(\bar{t}-t_i)}\sum_{j=1}^{J}\sum_{t=t_i}^{\bar{t}}(y_{ijt}^{obs}-\frac{1}{S}\sum_{i=1}^{S}y_{ijt}^{s}(\theta))\mathbb{1}\{n_{hj}^{g}<6\}\mathbb{1}\{AE_{jt,g}^{s,1}=0\}\Big\}=0. \tag{F.16}$$

This moment captures, for those foreign countries that are less than 6,000 km away from Costa Rica and have no country closer than 426 km to them, the difference between the export probability in the observed sample and the average export probability across S simulated samples. Similarly, the moment given by the function in equation (F.15b) for $(x_1, x_2) = (g, 1)$ is

$$\frac{1}{M}\sum_{i=1}^{M}\Big\{\frac{1}{J(\bar{t}-t_i)}\sum_{j=1}^{J}\sum_{t=t_i}^{\bar{t}}(y_{ijt}^{obs}-\frac{1}{S}\sum_{i=1}^{S}y_{ijt}^{s}(\theta))\mathbb{1}\{n_{hj}^{g}<6\}\mathbb{1}\{0<AE_{jt,g}^{s,1}\leqslant p_{66}(AE_{jt,g}^{s,1})\}\Big\}=0.$$

This moment captures, for foreign countries that are less than 6,000 km away from Costa Rica and have countries located less than 426 km away from them whose aggregate export potential is positive but below its 66th percentile, the difference between the export probability in the observed sample and the average export probability across S simulated samples.

Equation (F.15) lists six moments for each x_1 in $\{g, l, a\}$ and each x_2 in $\{1, 2, 3\}$. Thus, this block could include 54 moments in total. However, two of these 54 moments select empty subsets of countries. As a result, the second block includes 52 moments in total.

The third block includes moments that aim to identify the parameters of the distribution of the unobserved (to the researcher) terms ν_{it} and ω_{it}. That is, this block targets the identification of

$$(\sigma_\nu, p, \{(\gamma_x^N, \kappa_x^N)\}_{x=\{g,l,a\}}),$$

which enter the model through the expressions in equations (10) and (11). To help identify the variance of the fixed cost shock ν_{ijt}, σ_ν^2, we use moments defined by the following two functions

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}\sum_{i'\neq i} y_{i'jt}\mathbb{1}\{Q(r_{iht}) = Q(r_{i'ht})\}, \tag{F.17a}$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = \mathbb{1}\{\sum_{j=1} y_{ijt} > 0\}, \tag{F.17b}$$

where $Q(\cdot)\colon \mathbb{R}^+ \to \{1, 2, 3, 4\}$ is a function that maps the firm's domestic revenue to its corresponding quartile. The moment defined by the function in equation (F.17a) captures, on average across periods and pairs of firms i and i' whose domestic sales belong to the same quartile of the distribution, the similarity in the sets of export destinations. The function in equation (F.17b) captures whether firm i is an exporter at period t. These two moments help identify σ_ν.

With the aim of identifying p, we use moments defined by the following two functions

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}(1-y_{ijt-1})y_{ijt-2} + y_{ijt}(1-y_{ijt-1})(1-y_{ijt-2})y_{ijt-3}, \tag{F.18a}$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = (1-y_{ijt})y_{ijt-1}(1-y_{ijt-2}) + (1-y_{ijt})y_{ijt-1}y_{ijt-2}(1-y_{ijt-3}). \tag{F.18b}$$

The function in equation (F.18a) captures short (lasting one or two periods) spells outside of an export market. The function in equation (F.18b) captures short export spells. As firms have perfect foresight on all payoff-relevant variables other than the shock ω_{ijt}, short-lived transitions in and out of a market will be mostly driven by unexpected realizations of this fixed cost shock. The functions in equation (F.18) measure the frequency with which these short-lived spells take place.

Finally, with the aim of identifying $\{(\gamma_x^N, \kappa_x^N)\}_{x=\{g,l,a\}}$, we use moments defined by the functions

$$m_k(y,z,x) = y_{ijt} \sum_{j'=1}^{J} y_{ij't} \mathbb{1}\{y_{ijt-1} = y_{ij't-1}\} \mathbb{1}\{Q(E_{ijt}) = Q(E_{ij't})\} \mathbb{1}\{\underline{n}_{x_1}^{x_2} \leqslant n_{jj't}^{x_1} < \bar{n}_{x_1}^{x_2}\} \qquad \text{(F.19)}$$

for any x_1 in $\{g,l,a\}$ and any x_2 in $\{1,2,3\}$, where $Q(\cdot) : \mathbb{R}^+ \to \{1,2,3,4\}$ is a function that maps a country's export potential into its corresponding quartile. For any x_1 in $\{g,l,a\}$ and any x_2 in $\{1,2,3\}$, the thresholds $\underline{n}_{x_1}^{x_2}$ and $\bar{n}_{x_1}^{x_2}$ are determined as in equations (F.12) to (F.14). E.g., the moment built using the function in equation (F.19) for $(x_1,x_2) = (g,1)$ captures, on average across firms and periods, the frequency with which firms simultaneously export to any two countries j and j' in which they had the same export status in the previous period (as imposed by the condition that y_{ijt-1} and $y_{ij't-1}$ should coincide), that have similar export potentials (as imposed by the condition that E_{ijt} and $E_{ij't}$ should fall in the same quartile), and that are less than 426 km apart. Loosely, the function in equation (F.19) for $(x_1,x_2) = (g,1)$ captures the correlation in firm choices across countries of similar market size that are geographically close to each other.

The function in equation (F.19) for $(x_1,x_2) = (g,2)$ is analogous to that for $(x_1,x_2) = (g,1)$, differing only in that, instead of focusing on pairs of countries that are less than 426 km apart, it focuses on pairs of countries whose bilateral distance is larger than 426 km and smaller than 790 km. Similarly, the function in equation (F.19) for $(x_1,x_2) = (g,3)$ focuses on pairs of countries whose bilateral distance is larger than 790 km and smaller than 1,153 km. Thus, the functions in equation (F.19) for $x_1 = g$ and all three possible values of x_2 allow us to identify the parameters determining the correlation between ν_{ijt} and $\nu_{ij't}$ as a function of the geographical distance between countries j and j'.

Equations (F.17) and (F.18) list two moments each. Equation (F.19) lists one moment for each x_1 in $\{g,l,a\}$ and each x_2 in $\{1,2,3\}$. Thus, the third block includes 13 moments in total.

F.4 Additional Details on SMM Estimator

We provide here additional details on two aspects of our SMM estimator. In Appendix F.4.1, we describe how we compute the vector of simulated choices $y_i^s(\theta)$ that enter the moment conditions; see equation (26). In Appendix F.4.2, we describe how we compute our SMM estimates given the vector of moment conditions.

F.4.1 Computing Vector of Simulated Choices

Given a value of the vector θ of fixed and sunk cost parameters, we describe here the steps we follow to compute each of the moment conditions we use in our estimation.

First step. For each firm i in the sample, we take $S = 5$ draws of the vector of unobserved payoff-relevant variables χ_i defined in equation (F.6). Specifically, for each draw, we implement the following procedure.

First, if we observe firm i in the first sample year, $\underline{t}$, then we treat its birth year, $\underline{t}_i$, as unknown, and we draw it randomly from the empirical distribution of firm ages in Costa Rica in 2010, as reported in World Bank (2012). If we do not observe firm i in $\underline{t}$, then we assume its birth year coincides with the first year it appears in the sample. The firm's birth year is thus observed, and not randomly drawn, in this case.[27]

[27] A firm is in our dataset as long as it has positive domestic sales, regardless of whether it exports.

Second, we simulate $\ln(r_{iht})$ for every out-of-sample period in which the firm is active; i.e., for all t in $[\underline{t}_i, \underline{t}) \cup (\bar{t}, T]$. If $\underline{t}_i < \underline{t}$, we simulate $(\ln(r_{ih\underline{t}_i}), \ldots, \ln(r_{ih\underline{t}}))$ from a jointly normal distribution as determined by the corresponding AR(1) process for $\ln(r_{iht})$ specified in Section 3.3, conditioning on the firm's observed domestic sales in the first sample year, $r_{ih\underline{t}}$, as terminal condition, and on the the unconditional mean of this process as initial condition. To simulate $(\ln(r_{ih\bar{t}}), \ldots, \ln(r_{ihT}))$, we first draw $T - \bar{t} + 1$ independent standard normal variables, which we then multiply by σ_r. We then use these draws of e^r_{iht} for every t in $[\bar{t}+1, T]$, together with the firm's observed domestic sales in the last sample year, $r_{ih\bar{t}}$, to generate values of the firm's log domestic sales for every t in $[\bar{t}+1, T]$. In this case, $\ln(r_{ih\bar{t}})$ operates as an initial condition of the corresponding process.

Third, we draw firm i's fixed cost shocks ν_{ijt} and ω_{ijt} for every country $j = 1, \ldots, J$ and every t in $[\underline{t}_i, T]$. To obtain these draws of ν_{ijt}, we first draw $J(T - \underline{t}_i + 1)$ independent standard normal random variables, which we then multiply by the Cholesky decomposition of the variance matrix in equation (10). To obtain these draws of ω_{ijt}, we first draw $J(T - \underline{t}_i + 1)$ independent random variables distributed uniformly between 0 and 1; we then set $\omega_{ijt} = \underline{\omega}$ if the draw corresponding to country j and period t is smaller than the parameter p introduced in equation (11), and $\omega_{ijt} = \bar{\omega}$ otherwise.

Fourth, for each country j, we draw α_{jt} for every t between the earliest birth year in the corresponding simulated sample and the initial sample year, and for every t between the last sample year and the terminal period; i.e., for all t in $[\min_i\{\underline{t}_i\}, \underline{t}) \cup (\bar{t}, T]$. We simulate α_{jt} for all t in $[\min_i\{\underline{t}_i\}, \underline{t})$ from a jointly normal distribution as determined by the corresponding AR(1) process for α_{jt} specified in Section 3.3, conditioning on the unconditional mean of this process as initial condition, and on the observed value of α_{jt} in the first sample year, $\alpha_{j\underline{t}}$, as terminal condition. To simulate $(\alpha_{j\bar{t}+1}, \ldots, \alpha_{jT})$, we first draw $T - \bar{t} + 1$ independent standard normal variables, which we then multiply by σ_α. We then use these draws of e^α_{jt} for every t in $[\bar{t}+1, T]$, together with the value of α_{jt} observed in the last sample year, $\alpha_{j\bar{t}}$, to generate values of α_{jt} for every t in $[\bar{t}+1, T]$. In this case, $\alpha_{j\bar{t}}$ operates as an initial condition of the corresponding AR(1) process.

Second step. For each firm i in the sample, we use the S draws of χ_i generated according to the procedure described above, the vector z_i of observed payoff-relevant variables, and a value of the parameter vector θ, to compute the vector of model-implied firm i's optimal choices $y^s_i(\theta)$ for all $s = 1, \ldots, S$ simulated samples. We do so implementing the algorithm described in Section 4.

F.4.2 Computing SMM Estimates

Denote the vector of moment conditions as $\mathcal{M}(y^{obs}, Z, x; \theta) = (\mathbb{m}_1(y^{obs}, Z, x; \theta), \ldots, \mathbb{m}_K(y^{obs}, Z, x; \theta))'$ where

$$\mathbb{m}_k(y^{obs}, Z, x; \theta) = \frac{1}{M}\sum_{i=1}^{M}\Big\{\frac{1}{J(\bar{t} - t_i)}\sum_{j=1}^{J}\sum_{t=t_i}^{\bar{t}}\big\{m_k(y^{obs}_i, z_i, x) - \frac{1}{S}\sum_{i=1}^{S} m_k(y^s_i(\theta), z_i, x)\big\}\Big\},$$

with $y^{obs} = \{y^{obs}_i\}_{i=1}^M$ and $Z = \{z_i\}_{i=1}^M$. Given $\mathcal{M}(y^{obs}, Z, x; \theta)$ and a $K \times K$ positive semi-definite matrix W, we compute our SMM estimate of θ as the solution to the following problem

$$\min_\theta \mathcal{M}(y^{obs}, Z, x; \theta) W \mathcal{M}(y^{obs}, Z, x; \theta)'. \tag{F.20}$$

To solve this problem numerically, we use a two-step algorithm: first, we use the TikTak global optimizer proposed in Arnoud et al. (2019) with 5,000 starting points, using BOBYQA as the local optimizer; second, we polish the outcome of the global optimizer using a Subplex local optimizer.

In practice, we compute a two-stage SMM estimate. In the first stage, we obtain estimates of θ, which we denote as $\hat{\theta}_1$, minimizing the objective function in equation (F.20) for a diagonal weight matrix $W = W_1$ in which every diagonal element $k = 1, \ldots, K$ equals

$$W_{1,k} = \frac{1}{(\mathbb{m}_k^{obs}(y^{obs}, Z, x))^2}, \qquad \text{with} \qquad \mathbb{m}_k^{obs}(y^{obs}, Z, x) \equiv \frac{1}{M}\sum_{i=1}^{M}\Big\{\frac{1}{J(\bar{t}-t_i)}\sum_{j=1}^{J}\sum_{t=t_i}^{\bar{t}} m_k(y_i^{obs}, z_i, x)\Big\}.$$

In the second stage, we obtain estimates of θ, which we denote as $\hat{\theta}_2$, minimizing the function in equation (F.20) for a diagonal weight matrix $W = W_2$ in which every diagonal element $k = 1, \ldots, K$ equals $W_{2,k} = (\hat{V}_k(y^{obs}, Z, x; \hat{\theta}_1))^{-1}$, with $\hat{V}_k(y^{obs}, Z, x; \hat{\theta}_1)$ the clustered-robust variance of the moment $\mathcal{M}_k(y^{obs}, Z, x; \hat{\theta}_1)$, with each cluster defined as a firm-year combination (see Section 11 in Hansen and Lee, 2019, for details). We present heteroskedasticity-robust, clustered at the firm-year level, and clustered at the firm level, standard error estimates. We compute each of these applying the formulas in Section 11 of Hansen and Lee (2019), with the adjustment for simulation noise in Gourieroux et al. (1993).

F.5 Alternative Simulation Draws

We evaluate here the sensitivity of our estimates of the vector θ of fixed and sunk cost parameters to the set of $S = 5$ draws of χ_i (see equation (F.6)) we use to compute those estimates. We take 50 independent sets of 5 draws of χ_i and, for each of them, we compute a new SMM estimate of θ. For each parameter in θ, we compute a non-parametric density of the estimates obtained in the 50 sets of simulations, and report in Table F.2 the mode of this density as well as our baseline estimate; see Table F.4 for our baseline estimates. Our baseline estimates are generally close to the mode of the distribution of the estimates obtained for different draws of χ_i, the only exception being the estimate of γ_g^F, which is 25% smaller than the mode of the density of the corresponding estimates.

Table F.2: Sensitivity of Baseline SMM Estimates to Alternative Simulation Draws

Parameters	Baseline Estimates	Alternative Estimates	Parameters	Baseline Estimates	Alternative Estimates
γ_0^F	62.92	63.53	κ_l^E	5.40	5.53
γ_g^F	13.11	17.68	γ_a^E	3.32	3.29
γ_l^F	4.14	2.79	φ_a^E	1.21	1.26
γ_a^F	29.28	28.99	κ_a^E	6.85	6.68
γ_0^S	114.76	115.09	γ_g^N	0.64	0.66
γ_g^S	19.95	19.88	κ_g^N	0.05	0.10
γ_l^S	0.23	0.26	γ_l^N	0.15	0.15
γ_a^S	21.83	21.07	κ_l^N	4.54	4.60
γ_g^E	9.83	10.79	γ_a^N	0.06	0.06
φ_g^E	1.96	1.98	κ_a^N	2.61	2.57
κ_g^E	6.02	6.03	σ_ν	80.04	79.98
γ_l^E	0.98	1.06	p	0.72	0.72
φ_l^E	2.74	2.76			

Note: the number in the "Baseline Estimates" column is the estimate reported in Table F.4; that in the "Alternative Estimates" column is the mode of the non-parametric density of the estimates obtained when reestimating our model using 50 alternative sets of draws of χ_i^s.

F.6 Estimation Results: Additional Details

F.6.1 First-step Estimates: Potential Export Revenue Parameters

In Table F.3, we present estimates of the parameters affecting export revenues; see Section 3.3.

Table F.3: Estimates of Potential Export Revenue Parameters and Their Process

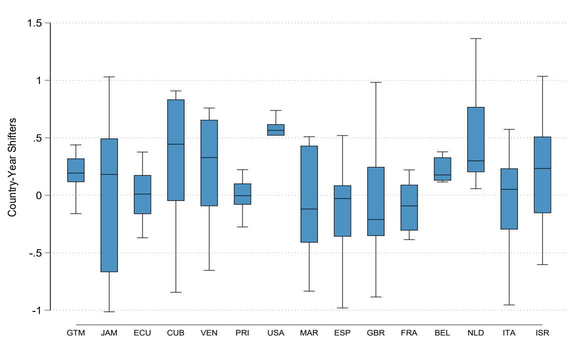
Potential Export Revenue Parameters		Process for Country- and Year-Specific Rev. Shifter		Process for Log Domestic Sales	
Parameter	Estimate (Standard Error)	Parameter	Estimate (Standard Error)	Parameter	Estimate (Standard Error)
α_y	1.856^a (0.066)	$\beta_{\alpha,g}$	-0.117^b (0.037)	ρ_r	0.857^a (0.012)
α_a	-3.832^a (0.066)	$\beta_{\alpha,l}$	-0.047 (0.071)	σ_r	0.865
α_r	0.285^a (0.041)	$\beta_{\alpha,a}$	-0.109 (0.079)		
		$\beta_{\alpha,gdp}$	0.079^a (0.019)		
		ρ_α	0.686^a (0.059)		
		σ_α	0.630		
Observations	13,293	Observations	467	Observations	43,300

Note: ^{a} denotes significance at 1%, ^{b} denotes significance at 5%. In parenthesis, standard error estimates. The results for *Potential Export Revenue Parameters* include country-year and sector fixed effects, and standard errors are heteroskedasticity robust. The results for *Process for Country- and Year-Specific Rev. Shifter* include no fixed effects, and standard errors are clustered by country. The results for *Process for Log Domestic Sales* include fixed effects for the firm's sector and province of location, and standard errors are clustered by firm.

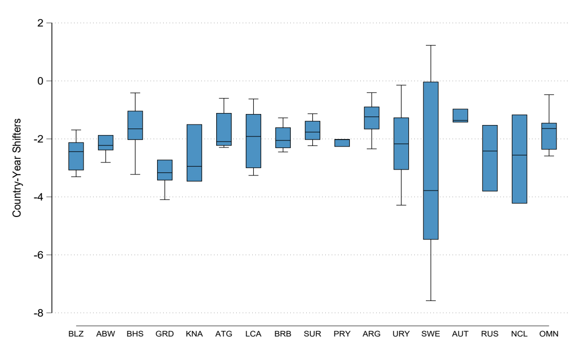
In Figure F.7, we present box plots summarizing the distribution of the estimated values of α_{jt} across all sample periods for several specific countries. Specifically, panels (a) and (b) contain information for the 15 countries with the largest and smallest median estimates of α_{jt}, respectively.

Figure F.7: Estimates of Country- and Year-Specific Export Revenue Shifters

(a) Top-15 Destinations



(b) Bottom-15 Destinations



Note: In both figures, countries are identified by their ISO 3166-1 alpha-3 code, and ordered in the horizontal axis by their distance to Costa Rica. For each country, the corresponding box plot represents (from top to bottom) the max, third quartile, median, first quartile and min of the estimated values of α_{jt} across all sample periods. Panel (a) displays box-plots of the estimates of $\{\alpha_{jt}\}_t$ for the 15 countries with the largest median estimates. Panel (b) displays analogous information for the 15 countries with the lowest median estimates.

F.6.2 Second-Step Estimates: Fixed and Sunk Costs Parameters

In Table F.4, we present estimates of the parameters entering fixed and sunk costs; see Section 3.4.

Table F.4: SMM Estimates of Fixed and Sunk Cost Parameters

Parameter	Estimate (Standard Errors)	Parameter	Estimate (Standard Errors)
γ_0^F	62.92^a (1.11)(1.34)(2.77)	κ_l^E	5.40 (6.05)(7.84)(19.56)
γ_g^F	13.11^a (0.38)(1.17)(3.43)	γ_a^E	3.32^a (0.04)(0.04)(0.06)
γ_l^F	4.14^a (0.99)(1.71)(4.71)	φ_a^E	1.21 (0.52)(0.73)(1.51)
γ_a^F	29.28^a (0.78)(0.62)(1.09)	κ_a^E	6.85^a (1.02)(1.48)(3.18)
γ_0^S	114.76^a (3.18)(3.09)(6.03)	γ_g^N	0.64^a (0.00)(0.00)(0.01)
γ_g^S	19.95^a (0.92)(1.10)(2.80)	κ_g^N	0.05^a (0.00)(0.00)(0.01)
γ_l^S	0.23 (3.56)(4.43)(8.36)	γ_l^N	0.15^a (0.00)(0.00)(0.01)
γ_a^S	21.83^a (1.04)(0.83)(1.46)	κ_l^N	4.54^a (0.29)(0.31)(0.50)
γ_g^E	9.83^a (2.33)(2.85)(6.42)	γ_a^N	0.06^a (0.01)(0.01)(0.01)
φ_g^E	1.96^a (0.50)(0.66)(1.55)	κ_a^N	2.61^a (0.00)(0.00)(0.00)
κ_g^E	6.02^a (0.28)(0.49)(0.66)	σ_ν	80.72^a (0.51)(0.79)(2.05)
γ_l^E	0.98^a (0.08)(0.07)(0.11)	p	0.72^a (0.00)(0.00)(0.00)
φ_l^E	2.74 (2.88)(3.79)(7.16)		

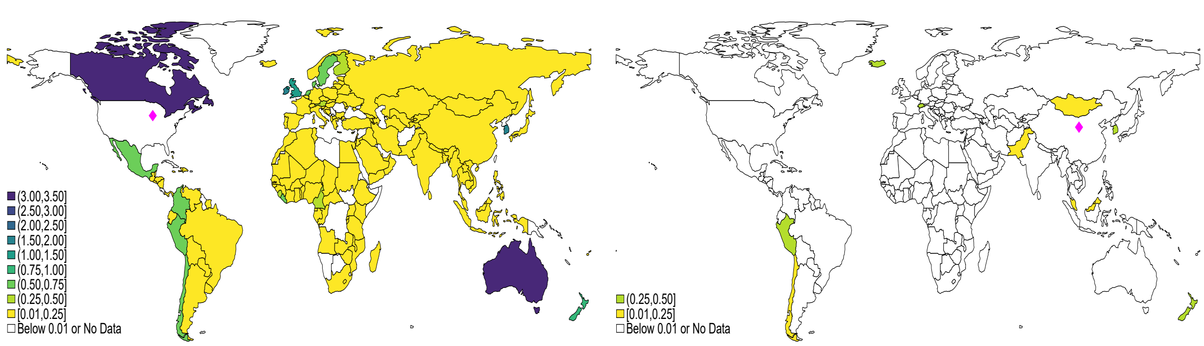
Note: ^{a} denotes significance at 1%. In parenthesis, robust standard errors, standard errors clustered by firm-year, and standard errors clustered by firm, respectively. Displayed markers of statistical significance are determined on the basis of the standard errors clustered by firm-year.

In Figure F.8, for the case of the US, China, Germany and Spain, we plot the value of $c_{jj't}/g_{jt}$ multiplied by 100 for all other destinations j'.

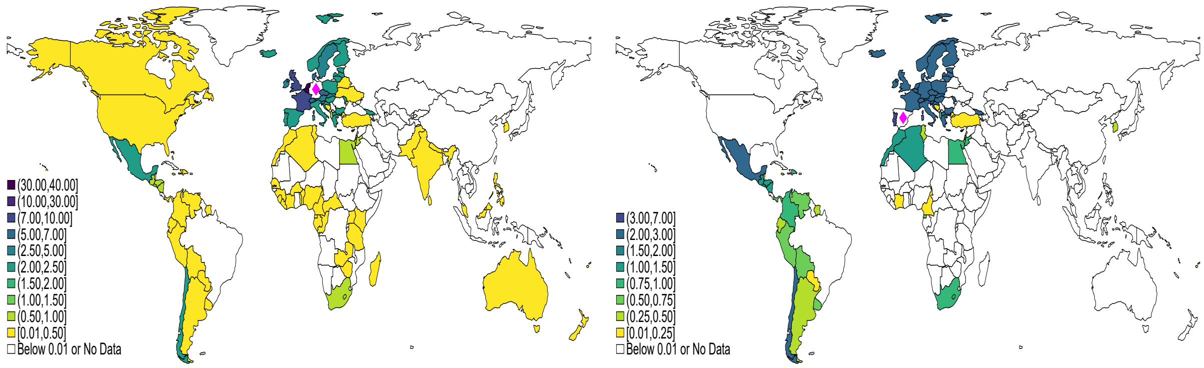
Figure F.8: Estimated Static Complementarities

(a) The United States

(b) China



(c) Germany (d) Spain

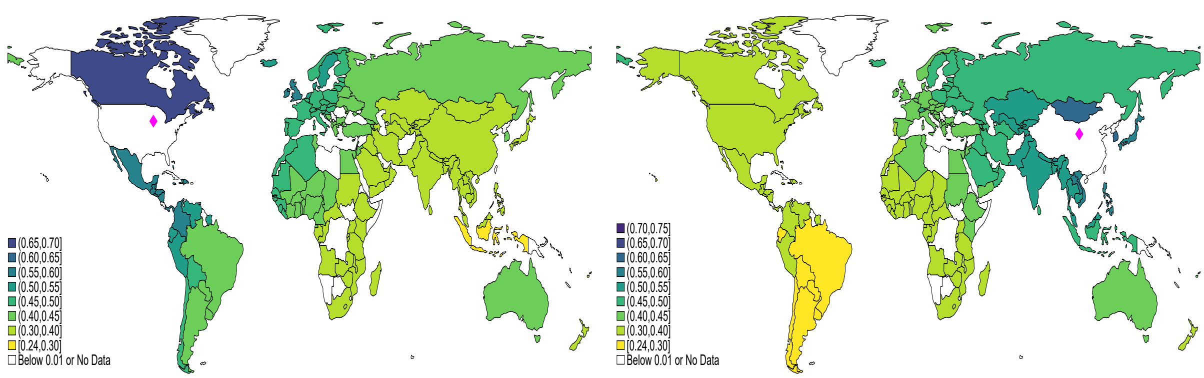


Note: In Panels (a), (b), (c) and (d) we illustrate, for the cases of the US, China, Germany, and Spain, respectively, the percentage reduction in fixed costs of exporting to these countries if the firm simultaneously also exports to each of the other possible export destinations.

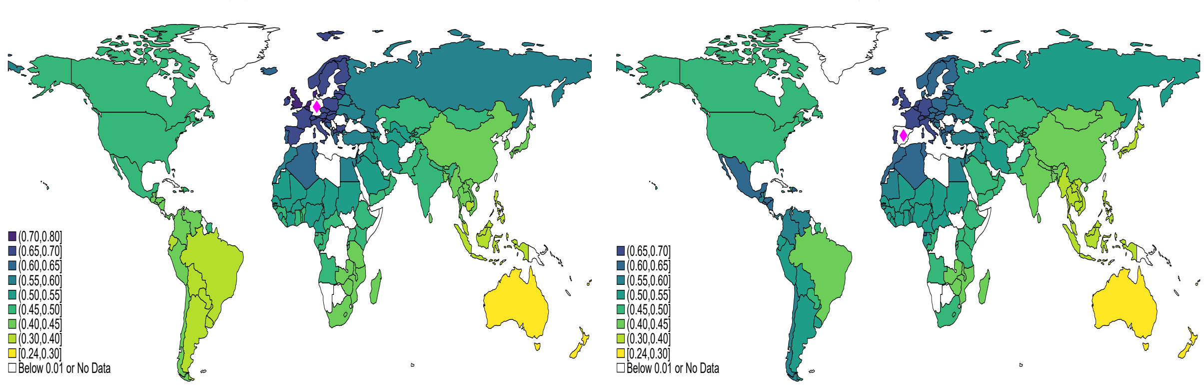
In Figure F.9, for the case of the US, China, Germany and Spain, we plot the value of $\rho_{jj't}$ for all other destinations j'.

Figure F.9: Estimated Pairwise Correlation Coefficients in Fixed Cost Shocks

(a) The United States (b) China



(c) Germany (d) Spain



Note: In Panels (a), (b), (c) and (d) we illustrate, for the cases of the US, China, Germany, and Spain, respectively, the correlation coefficient in the fixed cost shock ν_{ijt} between the corresponding country and every other country in the world.

F.7 Model Without Cross-Country Complementarities

We present here the estimates of a model analogous to that in Section 3 except for the restriction that the term in equation (9) equals zero for all countries and periods. Fixed and sunk costs in this restricted model thus only depend on the parameters $\theta_R \equiv (\gamma_0^F, \gamma_0^S, \sigma_\nu, p, \{(\gamma_x^F, \gamma_x^N, \kappa_x^N, \gamma_x^S)\}_{x=\{g,l,a\}})$.

In this restricted model, the estimation approach in Section 6.1 is still valid; thus, the estimates of the demand elasticity and the parameters entering potential export revenues coincide with those described in Section 6.1. To estimate θ_R, we follow an approach analogous to that in Section 5.2, using the same moments described in Section F.3. We present in Table F.5 the resulting estimates.

Table F.5: Fixed and Sunk Cost Parameter Estimates; Model Without Complementarities

Parameter	Estimate (Standard Errors)	Parameter	Estimate (Standard Errors)
γ_0^F	35.81^a (4.78)(7.93)(19.89)	γ_g^N	0.64^a (0.01)(0.01)(0.01)
γ_g^F	4.97^a (0.41)(0.75)(1.77)	κ_g^N	0.04^a (0.00)(0.00)(0.01)
γ_l^F	0.96 (2.64)(3.87)(9.59)	γ_l^N	0.18^a (0.03)(0.03)(0.07)
γ_a^F	6.32 (3.62)(6.05)(16.11)	κ_l^N	0.38 (0.52)(0.70)(1.59)
γ_0^S	70.70^a (6.17)(9.24)(21.09)	γ_a^N	0.10^a (0.01)(0.01)(0.03)
γ_g^S	36.21^a (0.22)(0.31)(0.31)	κ_a^N	0.42^a (0.05)(0.04)(0.10)
γ_l^S	0.16 (5.25)(9.93)(25.32)	σ_ν	41.59^a (0.76)(1.33)(3.36)
γ_a^S	27.39^a (4.48)(8.92)(24.36)	p	0.65^a (0.00)(0.00)(0.00)

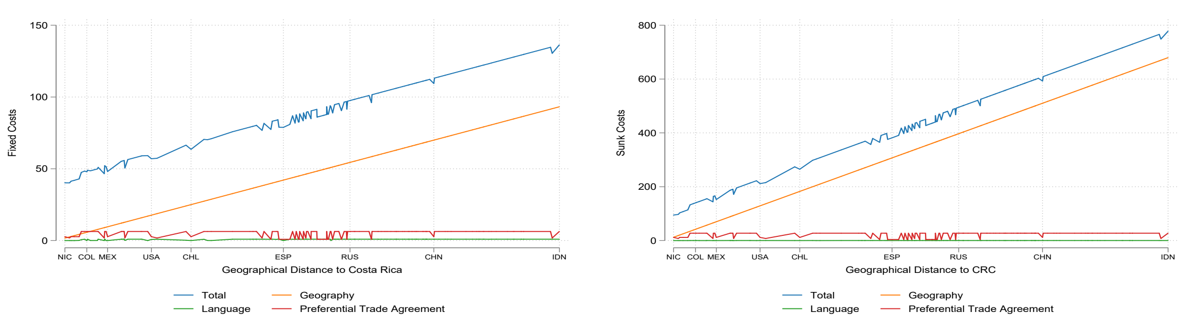
Note: ^{a} denotes significance at 1%. In parenthesis, robust standard errors, standard errors clustered by firm-year, and standard errors clustered by firm, respectively. Markers of statistical significance are determined on the basis of the standard errors clustered by firm-year.

Figure F.10 is analogous to Figure 1. The mean fixed cost function implied by the estimates in Table F.5 is smaller than the estimated mean fixed cost function for single-destination exporters

Figure F.10: Fixed and Sunk Cost Parameter Estimates; Model Without Complementarities

(a) Fixed Export Costs

(b) Sunk Export Costs



Note: In both figures, countries are identified by their ISO 3166-1 alpha-3 code, and placed in the horizontal axis by their distance to Costa Rica. The vertical axis indicates the estimated cost in thousands of 2010 USD.

displayed in panel (a) of Figure 1 for our general model with complementarities. This is to be expected, as the estimated mean fixed export costs in the restricted model without cross-country complementarities likely approximate a weighted average of the mean fixed export costs faced by different firms depending on their export bundles, with weights given by the frequency with which different export bundles are observed in the data.

F.8 Model With Permanent Unobserved Heterogeneity

We present here the estimates of a model analogous to that in Section 3 with the only difference that the fixed cost term ν_{ijt} introduced in equation (6) is not assumed to be independent over time but, instead, it is assumed to be permanent over time. That is, instead of assuming that $\nu_{ijt} \perp\!\!\!\perp \nu_{ijt'}$ for $t \neq t'$ (as imposed in the model described in Section 3 according to equation (10b)), we assume instead that $\nu_{ijt} = \nu_{ij}$. We present in Table F.6 the resulting estimates of all model parameters.

Table F.6: Estimates of Fixed and Sunk Cost Parameters in Model With $\nu_{ijt} = \nu_{ij}$

Parameter	Estimate (Standard Errors)	Parameter	Estimate (Standard Errors)
γ_0^F	87.07^a (0.57)(0.74)(1.79)	κ_l^E	4.54 (1.29)(2.99)(7.71)
γ_g^F	28.13^a (0.27)(0.32)(0.74)	γ_a^E	5.83^a (0.50)(0.72)(1.82)
γ_l^F	0.12 (0.77)(1.43)(4.02)	φ_a^E	1.30 (0.16)(0.28)(0.76)
γ_a^F	31.33^a (0.99)(1.56)(3.69)	κ_a^E	7.01^a (0.33)(0.46)(1.10)
γ_0^S	106.31^a (1.32)(1.71)(4.05)	γ_g^N	0.83^a (0.31)(0.54)(1.43)
γ_g^S	10.58^a (0.77)(1.00)(2.37)	κ_g^N	0.11^a (0.00)(0.00)(0.01)
γ_l^S	0.31 (2.36)(2.93)(6.76)	γ_l^N	0.04^a (0.00)(0.00)(0.01)
γ_a^S	33.78^a (1.92)(3.47)(9.40)	κ_l^N	4.60^a (0.51)(0.68)(1.62)
γ_g^E	10.52^a (0.04)(0.05)(0.10)	γ_a^N	0.06^a (0.01)(0.01)(0.01)
φ_g^E	2.57^a (0.16)(0.20)(0.41)	κ_a^N	2.30^a (0.45)(0.66)(1.75)
κ_g^E	9.11^a (0.28)(0.37)(0.79)	σ_ν	79.79^a (0.00)(0.00)(0.01)
γ_l^E	1.34^a (0.11)(0.14)(0.31)	p	0.67^a (0.00)(0.00)(0.00)
φ_l^E	3.48 (1.12)(1.41)(2.93)		

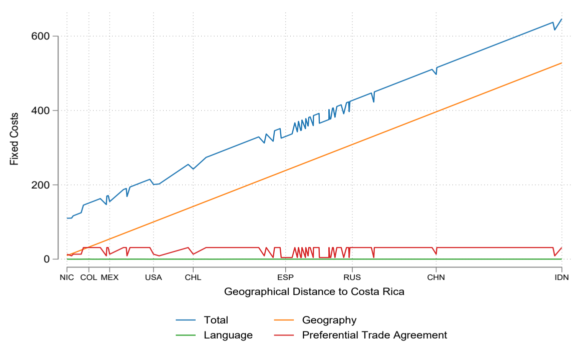
Note: [a] denotes significance at 1%. In parenthesis, robust standard errors, standard errors clustered by firm-year, and standard errors clustered by firm, respectively. Displayed markers of statistical significance are determined using the standard errors clustered by firm-year.

Figure F.11 is analogous to Figure 1. The sunk cost function implied by the estimates in Table F.6, displayed in panel (b) in Figure F.11, is smaller than the estimated sunk cost function displayed in panel (b) of Figure 1. There is a clear intuition for the negative impact that allowing for constant-over-time unobserved heterogeneity in fixed export costs has on the estimated sunk costs. This persistent heterogeneity in fixed export costs generates persistence in the firm's export status in any given destination. As a result, in the presence of persistent unobserved heterogeneity in fixed export costs, the estimated sunk costs need not be as large as in our baseline model in order to match the observed persistence in firms' export status in a destination.

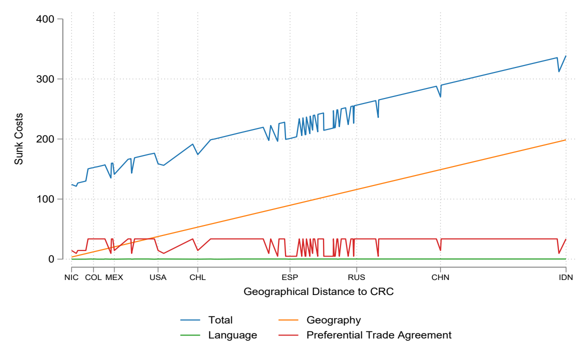
The estimated mean fixed cost function for single-destination exporters displayed in panel (a) of Figure F.11 is above the corresponding mean fixed cost function displayed in panel (a) of Figure 1. That is, in any given destination country, the estimated mean fixed export costs are larger in the model with persistent unobserved heterogeneity in fixed costs than in the model in which this heterogeneity is assumed to be independent over time. Intuitively, in order to rationalize the observed export entry patterns, the estimated fixed export costs must be larger in the model with smaller estimated sunk export costs.

Figure F.11: Fixed and Sunk Costs Estimates in Model With $\nu_{ijt} = \nu_{ij}$

(a) Fixed Export Costs



(b) Sunk Export Costs



Note: In both figures, countries are identified by their ISO 3166-1 alpha-3 code, and placed in the horizontal axis by their distance to Costa Rica. The vertical axis indicates the estimated cost in thousands of 2010 USD.

F.9 Goodness of Fit

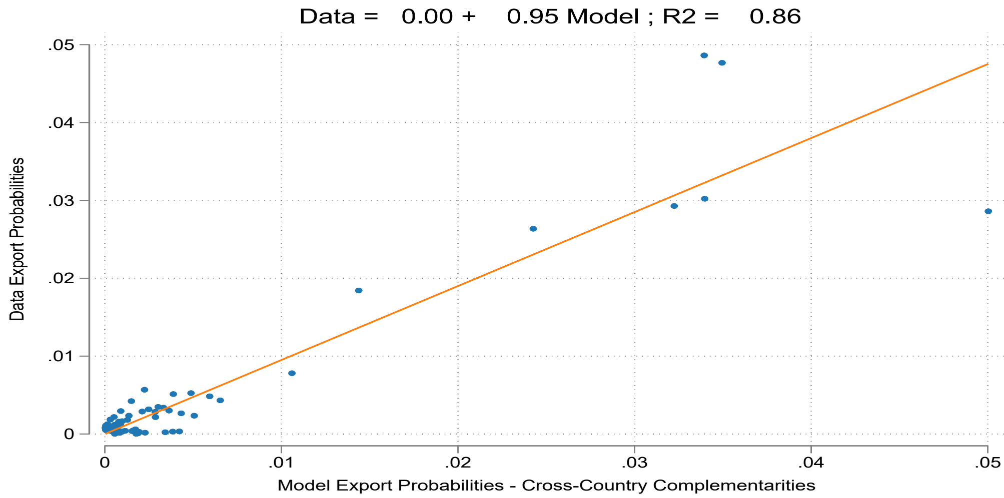
We present in Figure F.12 two plots that compare the observed share of firms in the sample that export to each possible destination country to the corresponding export probability predicted by our estimated model.

In panel (a), we present a scatter plot where each dot corresponds to a potential destination country, the coordinate in the horizontal axis corresponds to the model-predicted export probability, and the coordinate in the vertical axis corresponds to the observed share of firms exporting to the corresponding destination country. When regressing the observed export shares on the model-predicted export probabilities, we observe that the estimate of the regression constant equals zero, and the estimate of the regression slope equals 0.95. The R^2 for this regression is 0.86.

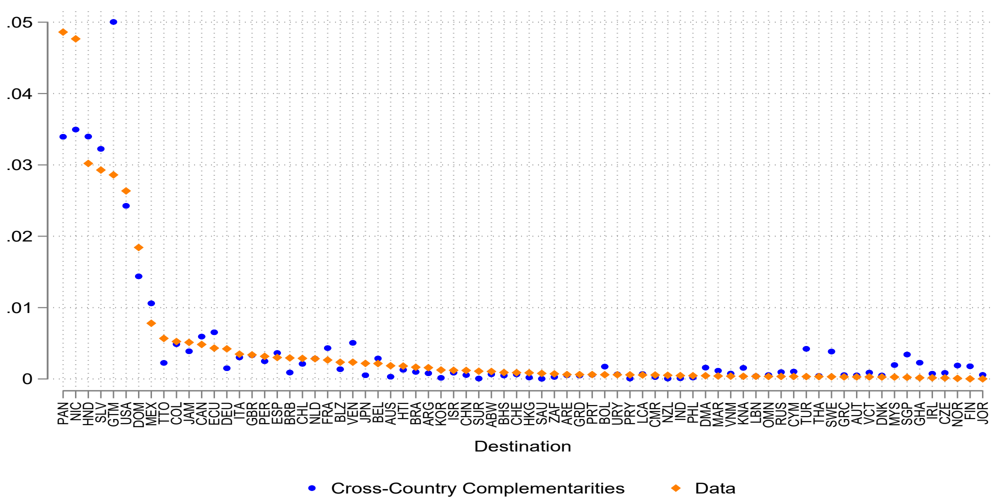
In panel (b), we list all possible destination countries in the horizontal axis and, for each of them, we display both the predicted export probability observed in the data (in orange diamonds) and that predicted by our estimate model (in blue circles). As the figure illustrates, the predicted export probabilities in our model align relatively well with the observed export probabilities observed in the data. Some of the largest mismatches are observed for Panama and Nicaragua; for these two countries, the model under-predicts the share of firms exporting to it. Panama and Nicaragua are the only two destination countries that share a border with Costa Rica and, consequently, the mismatch between observed and predicted export probabilities may be due to the impact that sharing a border with the firm's home country has on the fixed and sunk export costs the firm faces when exporting to it.

Figure F.12: Goodness-of-Fit Measures of Export Probabilities by Country

(a) Regression of Data on Model Implications



(b) Differences in Export Probabilities



Note: Panel (a) shows a scatter plot of the model predicted export shares by destination country (in the horizontal axis) against its observed value (in the vertical axis). The regression estimates and R^2 displayed on top of the figure are unrestricted. Panel (b) displays, for each possible destination country (listed in the horizontal axis), the predicted export probability observed in the data (in orange diamonds) and that predicted by our estimate model (in blue circles).

G Properties of Model With Complementarities

G.1 Impact of Complementarities Across Destinations

We consider here a simplified version of the model in Section 3 with the goal of understanding the role complementarities play in determining firm choices. We impose on the model in Section 3 the following restrictions: (a) there are two markets, A and B; (b) for both markets, the fixed cost gravity term g_{jt} and sunk costs s_{jt} are constant over time; (c) the complementarity term in fixed costs c_{ABt} is constant over time; (d) $\omega_{ijt} = 0$ for every i, j and t; (e) $\alpha_y = 0$ and all determinants of export revenues are constant over time, implying r_{ijt} is constant over time for every i and j.

Dropping the t subscript from all constant variables, and denoting the complementarities between markets A and B as c, firm i solves the following optimization problem at $t = 0$:

$$\max_{\{y_{jt}\}_{jt}} \sum_{t \geqslant 0} \left\{ \delta^t (y_{iAt}\pi_{iA} - (1 - y_{iAt-1})s_A + y_{iBt}\pi_{iB} - (1 - y_{iBt-1})s_B + y_{iAt}y_{iBt}c) \right\} \tag{G.1}$$

where, for any j, $\pi_{ij} = \eta^{-1}r_{ij} - g_j - \nu_{ij}$ is the potential export profits of firm i in j net of all components of fixed costs other than the complementarity term. As no firm can export before the first period of activity, it holds that $y_{iAt-1} = y_{iBt-1} = 0$ when $t = 0$.

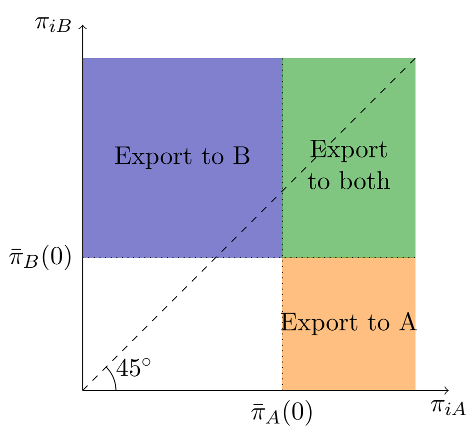
We consider two cases: one in which $c = 0$, and one in which $c > 0$. We keep all throughout the assumption that sunk export costs are lower in country B than in country A; i.e., $s_B < s_A$.

Case 1: no complementarities. In this case, $c = 0$ and the firm's export decision is independent across countries. As the problem in equation (G.1) is stationary, a firm exports to any country $j = \{A, B\}$ at any period $t \geqslant 0$ if and only if $\pi_{ij} \geqslant \bar{\pi}_j(0)$, for $\bar{\pi}_j(0) \equiv (1 - \delta)s_j$. Thus, as shown in panel (a) in Figure G.1, firms with $\pi_{iA} < \bar{\pi}_A$ and $\pi_{iB} \geqslant \bar{\pi}_B$ export only to B; firms with $\pi_{iA} \geqslant \bar{\pi}_A$ and $\pi_{iB} < \bar{\pi}_B$ export only to A; and, firms with $\pi_{iA} \geqslant \bar{\pi}_A$ and $\pi_{iB} \geqslant \bar{\pi}_B$ export to both countries. Consistently with the parametrization that $s_B < s_A$, the plot in panel (a) of Figure G.1 assumes that $\bar{\pi}_B(0) < \bar{\pi}_A(0)$.

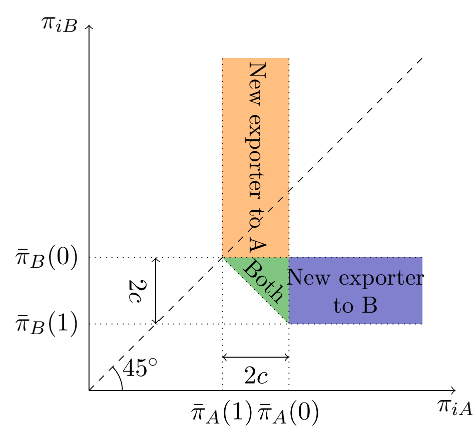
Case 2: positive complementarities. In this case, $c > 0$ and the firm's export decision is not independent across countries. Conditional on exporting to country $j' \neq j$, exporting to j is optimal if and only if $\pi_{ij} \geqslant \bar{\pi}_j(1)$ with $\bar{\pi}_j(1) = (1 - \beta)s_j - 2c$. Note that $\bar{\pi}_j(1) < \bar{\pi}_j(0)$ for any $c > 0$. Panel

Figure G.1: Export Choices Models With and Without Complementarities

(a) Model Without Complementarities



(b) Model With Complementarities



(b) in Figure G.1 illustrates the new exporters that emerge when c becomes positive. These new exporters are of two kinds. First, "natural exporters" to one of the markets (i.e., firms that export to one of the markets even when $c = 0$) and that, as complementarities become more important (i.e, as the value of c increases), start exporting to the other one. These are firms whose value of (π_{iA}, π_{iB}) falls in the orange and blue areas in panel (b). Second, firms that do not export when $c = 0$, but export to both markets when c increases. These are firms whose value of (π_{iA}, π_{iB}) falls in the green area in panel (b).

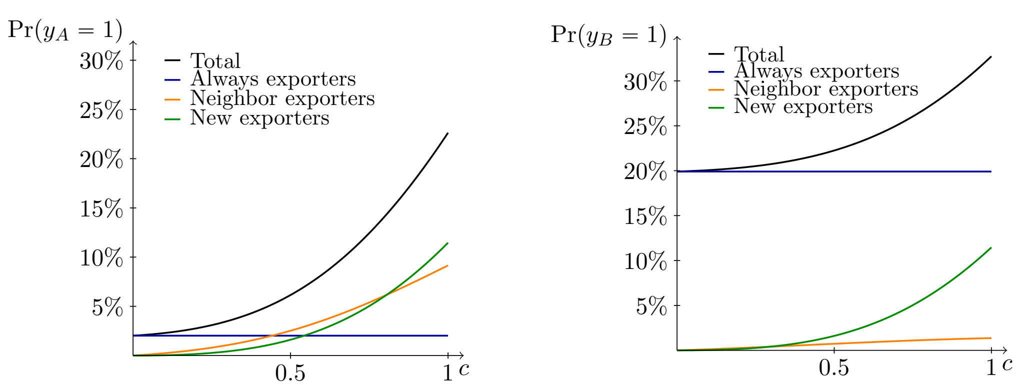
Panel (b) in Figure G.1 shows how a firm i, depending on the values of (π_{iA}, π_{iB}), changes its set of destinations when c switches from being equal to zero to being positive. How the share of firms exporting to either country changes as we change the value of c depends on the distribution of (π_{iA}, π_{iB}). In Figure G.2, we show how export shares change as we change the value of c when, for $j = \{A, B\}$, π_{ij} is normally distributed with mean μ (common in both markets) and variance equal to 1. We further assume that π_{iA} and π_{iB} are independent of each other. We impose values of μ, δ, s_A and s_B such that, when $c = 0$, the export share to A equals 2%, and the export share to B equals 20%. Thus, we can characterize markets A and B as being "small" and "large", respectively.

We extract several conclusions from Figure G.2. First, when comparing the export shares for positive values of c to those for $c = 0$, both the absolute and the relative increase in the export share is larger in the "small" export market (i.e., country A) than in the large one (i.e., country B). More specifically, when measuring the change in export shares as the value of c switches from zero to one, we observe that the percentage point increase in export shares in markets A and B is 21 pp. and 13 pp., respectively, and the relative increase in export shares in markets A and B is 11.5 (which equals 23%/2%) and 1.65 (which equals 33%/20%), respectively. Second, the reason for the larger impact of changes in c on export shares in A than in B is that there are many more firms that exported only to B in the case with $c = 0$ and add market A as export destination when c increases, than there are firms that exported only to A in the case with $c = 0$ and add market B as export destination when c increases; i.e., the probability that (π_{iA}, π_{iB}) is in the orange area in panel (b) of Figure G.1 is larger than the probability that it is in the blue area in the same graph.

Figure G.2: Export Share and Cross-Country Complementarities

(a) Small Market

(b) Large Market



Note: In panel (a), for each value of c, "Total" denotes the share of firms that export to A at that value of c; "Always exporters" denotes the share of firms that export to A at that value of c *and* also export to A when $c = 0$; "Neighbor exporters" denotes the share of firms that export to A at that value of c, do not export to A when $c = 0$, *and* export to B when $c = 0$; and "New exporters" denotes the share of firms that export to A at that value of c and export neither to A nor to B when $c = 0$. The interpretation of the labels for panel (b) is analogous.

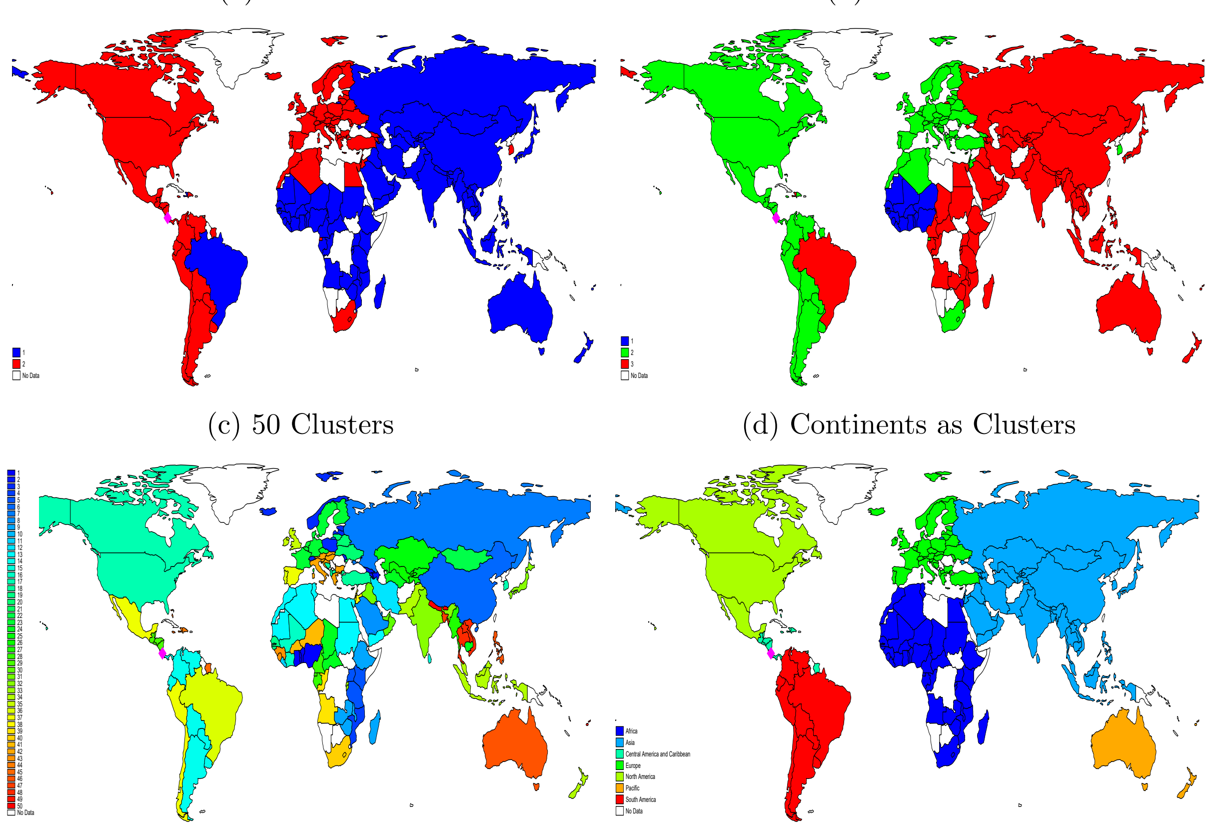
G.2 Clusters of Countries

We illustrate in Figure G.3 the clusters of countries we use when computing the results in Table 3. Given a fixed number of countries, we determine which countries to assign to each cluster following von Luxburg (2007). This procedure relies on an adjacency matrix between any two countries, which we compute using the formula in equation (9) and our estimates of $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$, and uses spectral clustering in order to categorize countries into groups.

Figure G.3: Alternative Clusters

(a) 2 Clusters (b) 3 Clusters

(c) 50 Clusters (d) Continents as Clusters



Note: the figures in the four panels illustrate the clusters of countries we use when computing the results in Table 3.

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